The data-driven COS method

Á. Leitao, C. W. Oosterlee, L. Ortiz-Gracia and S. M. Bohte

Delft University of Technology - Centrum Wiskunde & Informatica

Reading group, March 13, 2017

The COS method

- A lot of work behind: [FO08], [FO09], etc.
- Fourier-based method to price options.
- Starting point is risk-neutral valuation formula:

$$v(x,t) = \mathrm{e}^{-r(T-t)}\mathbb{E}\left[v(y,T)|x\right] = \mathrm{e}^{-r(T-t)}\int_{\mathbb{R}} v(y,T)f(y|x)\mathrm{d}y,$$

  where r is the risk-free rate and $f(y|x)$ is the density of the underlying process. Typically, we have:

$$x := \log\left(\frac{S(0)}{K}\right) \quad \text{and} \quad y := \log\left(\frac{S(T)}{K}\right),$$

- $f(y|x)$ is unknown in most of cases.
- However, characteristic function available for many models.
- Exploit the relation between the density and the characteristic function (Fourier pair).

The COS method - European options

- $f(y|x)$ is approximated, on a finite interval $[a, b]$, by a cosine series

$$f(y|x) = \frac{1}{b-a}\left(A_0 + 2\sum_{k=1}^{\infty} A_k(x)\cdot\cos\left(k\pi\frac{y-a}{b-a}\right)\right),$$

$$A_0 = 1,\;\; A_k(x) = \int_a^b f(y|x)\cos\left(k\pi\frac{y-a}{b-a}\right)\mathrm{d}y, \quad k = 1, 2, \ldots.$$

- Interchanging the summation and integration and introducing the definition

$$V_k := \frac{2}{b-a}\int_a^b v(y, T)\cos\left(k\pi\frac{y-a}{b-a}\right)\mathrm{d}y,$$

we find that the option value is given by

$$v(x, t) \approx \mathrm{e}^{-r(T-t)}{\sum_{k=0}^{\infty}}' A_k(x)V_k,$$

where $'$ indicates that the first term is divided by two.

Pricing European options with the COS method

- Coefficients A_k can be computed from the ChF.
- Coefficients V_k are known analytically (for many types of options).
- Closed-form expressions for the option Greeks Δ and Γ

$$\Delta = \frac{\partial v(x,t)}{\partial S} = \frac{1}{S(0)} \frac{\partial v(x,t)}{\partial x} \approx \exp(-r(T-t)) \sum_{k=0}^{\infty}{}' \frac{\partial A_k(x)}{\partial x} \frac{V_k}{S(0)},$$

$$\Gamma = \frac{\partial^2 v(x,t)}{\partial S^2} = \approx \exp(-r(T-t)) \sum_{k=0}^{\infty}{}' \left(-\frac{\partial A_k(x)}{\partial x)} + \frac{\partial^2 A_k(x)}{\partial x^2)} \right) \frac{V_k}{S^2(0)}$$

- Due to the rapid decay of the coefficients, $v(x,t)$, Δ and Γ can be approximated with high accuracy by truncating to N terms.

"Learning" densities

- *Statistical learning theory*: deals with the problem of finding a predictive function based on data.
- We follow the analysis about the problem of density estimation proposed by Vapnik in [Vap98].
- Given independent and identical distributed samples $X_1, X_2, \ldots, X_n$.
- By definition, density $f(x)$ is related to the *cumulative distribution function*, $F(x)$, by means of the expression

$$\int_{-\infty}^{x} f(y)\mathrm{d}y = F(x).$$

- Function $F(x)$ is approximated by the empirical approximation

$$F_n(x) = \frac{1}{n}\sum_{i=1}^{n} \eta(x - X_i),$$

where $\eta(\cdot)$ is the step-function. Convergence $\mathcal{O}(1/\sqrt{n})$.

Regularization approach

- The previous equation can be rewritten as a linear operator equation

$$Cf = F \approx F_n,$$

  where the operator $Ch := \int_{-\infty}^{x} h(z)\mathrm{d}z$.
- Stochastic ill-posed problem. *Regularization method* (Vapnik).
- Given a lower semi-continuous functional $W(f)$ such that:
  - Solution of $Cf = F_n$ belongs to $\mathcal{D}$, the domain of definition of $W(f)$.
  - The functional $W(f)$ takes real non-negative values in $\mathcal{D}$.
  - The set $\mathcal{M}_c = \{f : W(f) \leq c\}$ is compact in $\mathcal{H}$ (the space where the solution exits and is unique).
- Then we can construct the functional

$$R_{\gamma_n}(f, F_n) = L^2_{\mathcal{H}}(Cf, F_n) + \gamma_n W(f),$$

  where $L_{\mathcal{H}}$ is a metric of the space $\mathcal{H}$ (loss function) and γ_n is the parameter of regularization satisfying that $\gamma_n \to 0$ as $n \to \infty$.
- Under these conditions, a function f_n minimizing the functional converges almost surely to the desired one.

Regularization and Fourier-based density estimators

- Assume $f(x)$ belongs to the functions whose p-th derivatives belong to $L_2(0,\pi)$, the kernel $\mathcal{K}(z-x)$ and

$$W(f) = \int_{\mathbb{R}} \left(\int_{\mathbb{R}} \mathcal{K}(z-x) f(x) \mathrm{d}x \right)^2 \mathrm{d}z,$$

- The risk functional

$$R_{\gamma_n}(f, F_n) = \int_{\mathbb{R}} \left(\int_0^x f(y)\mathrm{d}y - F_n(x) \right)^2 \mathrm{d}x + \gamma_n \int_{\mathbb{R}} \left(\int_{\mathbb{R}} \mathcal{K}(z-x) f(x) \mathrm{d}x \right)^2 \mathrm{d}z.$$

- Denoting by $\hat{f}(u)$, $\hat{F}_n(u)$ and $\hat{\mathcal{K}}(u)$ the Fourier transforms, by definition

$$\begin{aligned}
\hat{F}_n(u) &= \frac{1}{2\pi} \int_{\mathbb{R}} F_n(x) \mathrm{e}^{-iux} \mathrm{d}x \\
&= \frac{1}{2n\pi} \int_{\mathbb{R}} \sum_{j=1}^{n} \eta(x - X_j) \mathrm{e}^{-iux} \mathrm{d}x = \frac{1}{n} \sum_{j=1}^{n} \frac{\exp(-iuX_j)}{iu},
\end{aligned}$$

where $i = \sqrt{-1}$ is the imaginary unit.

Regularization and Fourier-based density estimators

- By employing the *convolution theorem* and *Parseval's identity*

$$R_{\gamma_n}(f, F_n) = \left\| \frac{\hat{f}(u) - \frac{1}{n}\sum_{j=1}^{n} \exp(-iuX_j)}{iu} \right\|_{L_2}^2 + \gamma_n \left\| \hat{\mathcal{K}}(u)\hat{f}(u) \right\|_{L_2}^2 .$$

- The condition to minimize $R_{\gamma_n}(f, F_n)$ is given by,

$$\frac{\hat{f}(u)}{u^2} - \frac{1}{nu^2}\sum_{j=1}^{n} \exp(-iuX_j) + \gamma_n \hat{\mathcal{K}}(u)\hat{\mathcal{K}}(-u)\hat{f}(u) = 0,$$

which gives us,

$$\hat{f}_n(u) = \left(\frac{1}{1 + \gamma_n u^2 \hat{\mathcal{K}}(u)\hat{\mathcal{K}}(-u)} \right) \frac{1}{n} \sum_{j=1}^{n} \exp(-iuX_j).$$

Regularization and Fourier-based density estimators

- $\mathcal{K}(x) = \delta^{(p)}(x)$, and the desired PDF, $f(x)$ and its p-th derivative ($p \geq 0$) belongs to $L_2(0,\pi)$, the risk functional becomes

$$R_{\gamma_n}(f, F_n) = \int_0^\pi \left(\int_0^x f(y)\mathrm{d}y - F_n(x) \right)^2 \mathrm{d}x + \gamma_n \int_0^\pi \left(f^{(p)}(x) \right)^2 \mathrm{d}x.$$

- Given *orthonormal functions*, $\psi_1(\theta), \ldots, \psi_k(\theta), \ldots$

$$f_n(\theta) = \frac{1}{\pi} + \frac{2}{\pi} \sum_{k=1}^{\infty} \tilde{A}_k \psi_k(\theta),$$

  with $\tilde{A}_0, \tilde{A}_1, \ldots, \tilde{A}_k, \ldots$ expansion coefficients, $\tilde{A}_k =< f_n, \psi_k >$.

- The coefficients $\tilde{A}_k$ cannot be directly computed from f_n, but

$$\begin{aligned}\tilde{A}_k &=< f_n, \psi_k >=< \hat{f}_n, \hat{\psi}_k > \\ &= \int_0^\pi \left(\left(\frac{1}{1 + \gamma_n u^2 \hat{\mathcal{K}}(u) \hat{\mathcal{K}}(-u)} \right) \frac{1}{n} \sum_{j=1}^{n} \exp(-iu\theta_j) \right) \cdot \hat{\psi}_k(u) \mathrm{d}u.\end{aligned}$$

Regularization and Fourier-based density estimators

- Using cosine series expansions, i.e., $\psi_k(\theta) = \cos(k\theta)$, it is well-known that
$$\hat{\psi}_k(u) = \frac{1}{2}(\delta(u-k) + \delta(u+k)).$$
- This facilitates the computation of $\tilde{A}_k$ avoiding the calculation of the integral. Thus, the minimum of R_{γ_n}

$$\tilde{A}_k = \frac{1}{2n}\left(\left(\frac{1}{1+\gamma_n(-k)^2\hat{\mathcal{K}}(-k)\hat{\mathcal{K}}(k)}\right)\sum_{j=1}^{n}\exp(ik\theta_j)\right.$$
$$\left. + \left(\frac{1}{1+\gamma_n k^2\hat{\mathcal{K}}(k)\hat{\mathcal{K}}(-k)}\right)\sum_{j=1}^{n}\exp(-ik\theta_j)\right)$$
$$= \frac{1}{1+\gamma_n k^2\hat{\mathcal{K}}(k)\hat{\mathcal{K}}(-k)}\frac{1}{n}\sum_{j=1}^{n}\cos(k\theta_j) = \frac{1}{1+\gamma_n k^{2(p+1)}}\frac{1}{n}\sum_{j=1}^{n}\cos(k\theta_j),$$

where $\theta_j \in (0,\pi)$ are given samples of the unknown distribution. In the last step, $\hat{\mathcal{K}}(u) = (iu)^p$ is used.

The data-driven COS method

- Employ the solution of the regularization problem for density estimation in the COS framework.
- In both, the density is assumed to be in the form of a cosine series expansion.
- The minimum of the functional is in terms of the expansion coefficients.
- Take advantage of the COS machinery: pricing options, Greeks, etc.
- The samples must follow risk-neutral measure (Monte Carlo paths).

The data-driven COS method

- Key idea: $\tilde{A}_k$ approximates A_k.
- Risk neutral samples from an asset at time T, $S_1(t), S_2(t), \dots, S_n(t)$.
- With a logarithmic transformation, we have

$$Y_j := \log\left(\frac{S_j(T)}{K}\right).$$

- The regularization solution is defined in $(0, \pi)$, by transformation

$$\theta_j = \pi \frac{Y_j - a}{b - a},$$

where the boundaries a and b are defined as

$$a := \min_{1 \le j \le n} (Y_j), \quad b := \max_{1 \le j \le n} (Y_j).$$

The data-driven COS method - European options

- The A_k coefficients are replaced by the data-driven $\tilde{A}_k$

$$A_k \approx \tilde{A}_k = \frac{\frac{1}{n}\sum_{j=1}^{n} \cos\left(k\pi \frac{Y_j - a}{b-a}\right)}{1 + \gamma_n k^{2(p+1)}}.$$

- The ddCOS pricing formula for European options

$$\tilde{v}(x,t) = \mathrm{e}^{-r(T-t)} {\sum_{k=0}^{\infty}}' \frac{\frac{1}{n}\sum_{j=1}^{n} \cos\left(k\pi \frac{Y_j - a}{b-a}\right)}{1 + \gamma_n k^{2(p+1)}} \cdot V_k$$
$$= \mathrm{e}^{-r(T-t)} {\sum_{k=0}^{\infty}}' \tilde{A}_k V_k.$$

- As in the original COS method, we must truncate the infinite sum to a finite number of terms N

$$\tilde{v}(x,t) = \mathrm{e}^{-r(T-t)} {\sum_{k=0}^{N}}' \tilde{A}_k V_k,$$

The data-driven COS method - Greeks

- Data-driven expressions for the Δ and Γ sensitivities.
- Define the corresponding sine coefficients as

$$\tilde{B}_k := \frac{\frac{1}{n}\sum_{j=1}^{n} \sin\left(k\pi \frac{Y_j - a}{b-a}\right)}{1 + \gamma_n k^{2(p+1)}}.$$

- Taking derivatives of the ddCOS pricing formulat w.r.t the samples, Y_j, the data-driven Greeks, $\tilde{\Delta}$ and $\tilde{\Gamma}$, can be obtained by

$$\tilde{\Delta} = \mathrm{e}^{-r(T-t)} {\sum_{k=0}^{N}}' \tilde{B}_k \cdot \left(-\frac{k\pi}{b-a}\right) \cdot \frac{V_k}{S(0)},$$

$$\tilde{\Gamma} = \mathrm{e}^{-r(T-t)} {\sum_{k=0}^{N}}' \left(\tilde{B}_k \cdot \frac{k\pi}{b-a} - \tilde{A}_k \cdot \left(\frac{k\pi}{b-a}\right)^2\right) \cdot \frac{V_k}{S^2(0)}.$$

The data-driven COS method - Variance reduction

- The ddCOS method admits in the computation of $\tilde{A}_k$.
- Here, *antithetic variates (AV)* to our method. Since the samples must be i.i.d., an immediate application of AV is not possible.
- Assume antithetic samples, Y_i', that can be computed without computational effort, a new estimator

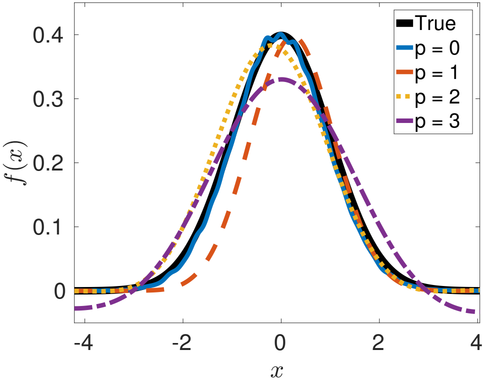
$$\bar{A}_k := \frac{1}{2}\left(\tilde{A}_k + \tilde{A}_k'\right),$$

  where $\tilde{A}_k'$ are "antithetic coefficients", obtained from Y_i'.
- It can be proved that the use of $\bar{A}_k$ results in a variance reduction.
- Additional information to reduce the variance. For example, the martingale property

$$\begin{aligned} S(T) &= S(T) - \frac{1}{n}\sum_{j=1}^{n} S_j(T) + \mathbb{E}[S(T)], \\ &= S(T) - \frac{1}{n}\sum_{j=1}^{n} S_j(T) + S(0)\exp(rT). \end{aligned}$$

Choice of parameters in ddCOS method

- The choice of optimal values of γ_n and p.
- There is no rule or procedure to obtain an optimal p.
- As a rule of thumb, $p = 0$ seems to be the most appropriate value.
- Fixing p, we rely on the computation of an optimal γ_n.



Choice of γ_n

- γ_n impacts the efficiency of the ddCOS method: it is related to the number of samples, n, and number of terms, N.
- For the regularization parameter γ_n, a rule that ensures asymptotic convergence
$$\gamma_n = \frac{\log \log n}{n}.$$
- In practical situations: not optimal.
- Exploit the relation between the empirical and real (unknown) CDFs.

Choice of γ_n

- This relation can be modeled by *statistical laws* or *statistics*: Kolmogorov-Smirnov, Anderson-Darling, Smirnov-Cramér–von Mises.
- Preferable: a measure of the distance between the $F_n(x)$ and $F(x)$ follows a known distribution.
- We have chosen Smirnov-Cramér–von Mises(SCvM):

$$\omega^2 = n \int_{\mathbb{R}} (F(x) - F_n(x))^2 \, \mathrm{d}F(x).$$

- Assume we have an approximation, F_{γ_n} (which depends on γ_n).
- An *almost* optimal γ_n is computed by solving the equation

$$\sum_{i=1}^{n} \left(F_{\gamma_n}(\bar{X}_i) - \frac{i-0.5}{n} \right)^2 = m_S - \frac{i}{12n},$$

where $\bar{X}_1, \bar{X}_2, \ldots, \bar{X}_n$ is the ordered array of samples $X_1, X_2, \ldots, X_n$ and m_S the mean of the ω^2.

Influence of γ_n

- To assess the impact of γ_n: *Mean integrated Squared Error* (MiSE):

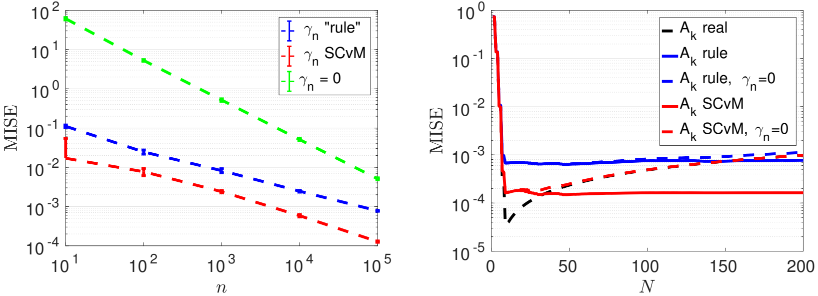
$$\mathbb{E}\left[\|f_n - f\|_2^2\right] = \mathbb{E}\left[\int_{\mathbb{R}} (f_n(x) - f(x))^2 \, \mathrm{d}x\right].$$

- A formula for the MiSE formula is derived in our context:

$$\mathrm{MISE} = \frac{1}{n}\sum_{k=1}^{N} \frac{1}{\left(1+\gamma_n k^{2(p+1)}\right)^2}\left(\frac{1}{2}+\frac{1}{2}A_{2k} - A_k^2\right) + \sum_{k=N+1}^{\infty} A_k^2.$$

- Two main aspects influenced γ_n: accuracy in n and stability in N.
- The quality of the approximated density can be also affected.

Influence of γ_n



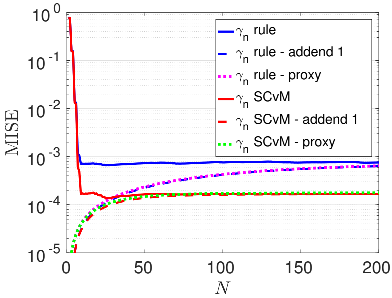
(a) Convergence in terms of n (b) Convergence in terms of N

Figure: Influence of γ_n: .

Optimal number of terms N

- Try to find a *minimum optimal* value of N.
- N considerably affects the performance.
- We wish to avoid the computation of any $\hat{A}_k$.
- We define a proxy for the MiSE and follow:

$$\text{MiSE} \approx \frac{1}{n} \sum_{k=1}^{N} \frac{\frac{1}{2}}{\left(1 + \gamma_n k^{2(p+1)}\right)^2}.$$



Optimal number of terms N

Data: n, γ_n
$N_{min} = 5$
$N_{max} = \infty$
$\epsilon = \frac{1}{\sqrt{n}}$
$\text{MiSE}_{prev} = \infty$
for $N = N_{min} : N_{max}$ **do**
    $\text{MiSE}_N = \frac{1}{n}\sum_{k=1}^{N} \frac{\frac{1}{2}}{\left(1+\gamma_n k^{2(p+1)}\right)^2}$
    $\epsilon_N = \frac{|\text{MiSE}_N - \text{MiSE}_{prev}|}{|\text{MiSE}_N|}$
    if $\epsilon_N > \epsilon$ **then**
        $N_{op} = N$
    else
        Break
    $\text{MiSE}_{prev} = \text{MiSE}_N$

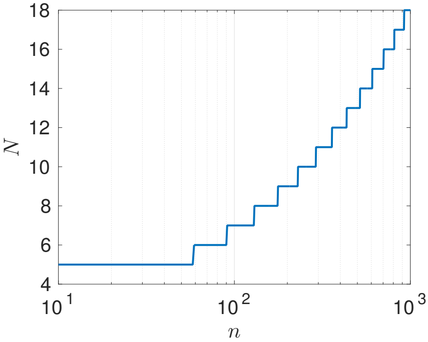


Figure: Almost optimal N.

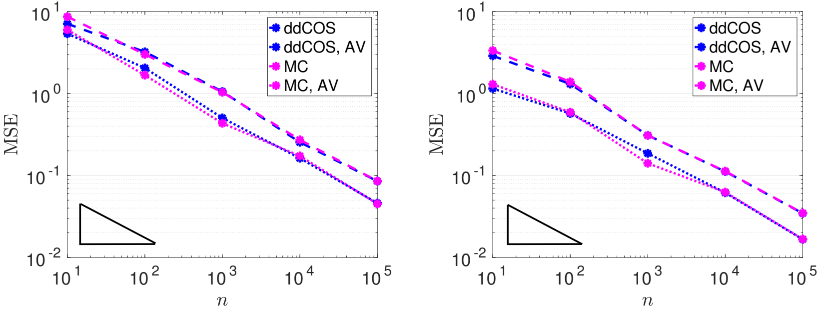
Applications of the ddCOS method

- Pricing options (no better than Monte Carlo).
- Sensitivities or Greeks.
- Models without analytic characteristic function. SABR model.
- Risk measures: VaR and Expected shortfall.
- Combinations.

Applications of the ddCOS method

- Unfortunately, the γ_n based on the SCvM statistic does not provide any benefit.
- The use of the γ_n rule entails faster ddCOS estimators.
- ddCOS converges with the expected convergence rate $\mathcal{O}(1/\sqrt{n})$.
- The variance reduction techniques are successfully applied.
- In Greeks computation, Monte Carlo-based methods may require one or two extra simulations.
- In the convergence tests, the reported values are computed as the average of 50 experiments.

Applications of the ddCOS - Option pricing

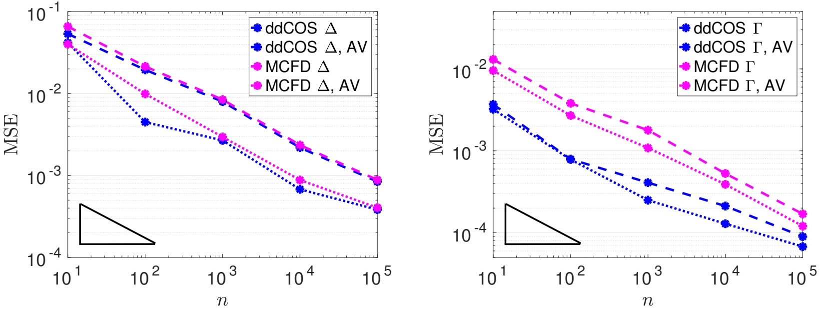


(a) Call: Strike $K = 100$.

(b) Put: Strike $K = 100$.

Figure: Convergence in prices of the ddCOS method: Antithetic Variates (AV); GBM, $S(0) = 100$, $r = 0.1$, $\sigma = 0.3$ and $T = 2$.

Applications of the ddCOS - Greeks estimation



(a) Δ (Call): Strike $K = 100$.

(b) Γ: Strike $K = 100$.

Figure: Convergence in Greeks of the ddCOS method: Antithetic Variates (AV); GBM, $S(0) = 100$, $r = 0.1$, $\sigma = 0.3$ and $T = 2$.

Applications of the ddCOS - Greeks estimation

K (% of $S(0)$)	80%	90%	100%	110%	120%
0.1			Δ		
Ref.	0.8868	0.8243	0.7529	0.6768	0.6002
ddCOS	0.8867	0.8240	0.7528	0.6769	0.6002
RE	1.1012×10^{-4}				
MCFD	0.8876	0.8247	0.7534	0.6773	0.6006
RE	7.5168×10^{-4}				
			Γ		
Ref.	0.0045	0.0061	0.0074	0.0085	0.0091
ddCOS	0.0045	0.0062	0.0075	0.0084	0.0090
RE	8.5423×10^{-3}				
MCFD	0.0045	0.0059	0.0071	0.0079	0.0083
RE	4.9554×10^{-2}				

Table: GBM call option Greeks: $S(0) = 100$, $r = 0.1$, $\sigma = 0.3$ and $T = 2$.

Applications of the ddCOS - Greeks estimation

K (% of $S(0)$)	80%	90%	100%	110%	120%
			Δ		
Ref.	0.8385	0.8114	0.7847	0.7584	0.7328
ddCOS	0.8383	0.8113	0.7846	0.7585	0.7333
RE	2.7155×10^{-4}				
MCFD	0.8387	0.8118	0.7850	0.7586	0.7330
RE	3.1265×10^{-4}				
			Γ		
Ref.	0.0022	0.0024	0.0027	0.0029	0.0030
ddCOS	0.0022	0.0024	0.0027	0.0029	0.0030
RE	8.2711×10^{-3}				
MCFD	0.0023	0.0026	0.0028	0.0031	0.0033
RE	6.118×10^{-2}				

Table: Merton jump-diffusion call option Greeks: $S(0) = 100$, $r = 0.1$, $\sigma = 0.3$, $\mu_j = -0.2$, $\sigma_j = 0.2$ and $\lambda = 8$ and $T = 2$.

Applications of the ddCOS - Greeks estimation

K (% of $S(0)$)	80%	90%	100%	110%	120%
			Δ		
Ref.	0.9914	0.9284	0.5371	0.0720	0.0058
ddCOS	0.9916	0.9282	0.5363	0.0732	0.0058
RE	5.2775×10^{-3}				
MCFD	0.9911	0.9279	0.5368	0.0737	0.0058
RE	5.5039×10^{-3}				

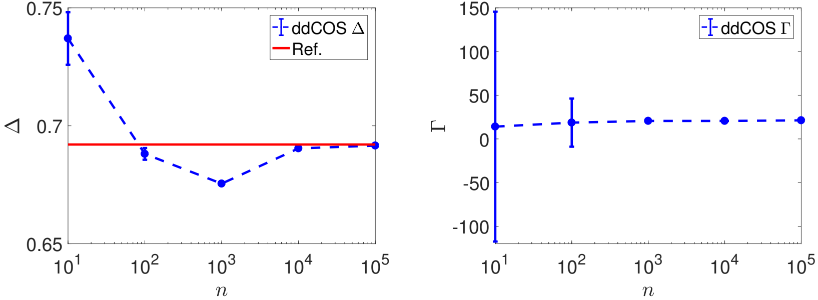
Table: Call option Greek Δ under the SABR model: $S(0) = 100$, $r = 0$, $\sigma_0 = 0.3$, $\alpha = 0.4$, $\beta = 0.6$, $\rho = -0.25$ and $T = 2$.

Applications of the ddCOS - Greeks estimation

K (% of $S(0)$)	80%	90%	100%	110%	120%
			Δ		
Ref.	0.8384	0.7728	0.6931	0.6027	0.5086
ddCOS	0.8364	0.7703	0.6902	0.6006	0.5084
RE	2.7855×10^{-3}				
Hagan	0.8577	0.7955	0.7170	0.6249	0.5265
RE	3.1751×10^{-2}				

Table: Δ under SABR model. Setting: Call, $S(0) = 0.04$, $r = 0.0$, $\sigma_0 = 0.4$, $\alpha = 0.8$, $\beta = 1.0$, $\rho = -0.5$ and $T = 2$.

Applications of the ddCOS - Greeks estimation



(a) Δ: Strike $K = 0.04$.

(b) Γ: Strike $K = 0.04$.

Figure: The ddCOS method: Greeks convergence test.

- In the context of the Delta-Gamma approach (COS in [OGO14]).
- The change in a portfolio value can be generalized.

$$L := -\Delta V = V(S,t) - V(S+\Delta S, t+\Delta t).$$

- The formal definition of the VaR reads

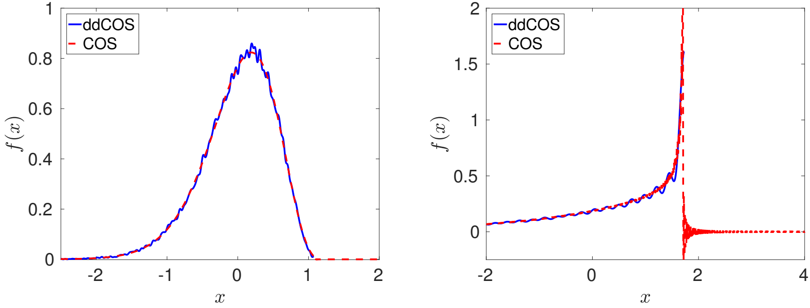
$$\mathbb{P}(\Delta V < \mathrm{VaR}(q)) = 1 - F_L(\mathrm{VaR}(q)) = q,$$

with q a predefined confidence level.
- Given the VaR, the ES measure is computed as

$$\mathrm{ES} := \mathbb{E}[\Delta V | \Delta V > \mathrm{VaR}(q)].$$

- Two portfolios with the same composition: one European call and half a European put on the same asset, maturity 60 days and $K = 101$.
- Different time horizons: 1 day (Portfolio 1) and 10 days (Portfolio 2). The asset follows a GBM with $S(0) = 100$, $r = 0.1$ and $\sigma = 0.3$.

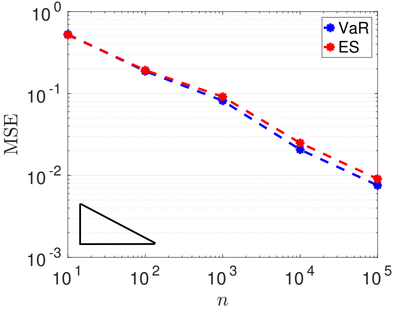
Applications of the ddCOS - Risk measures



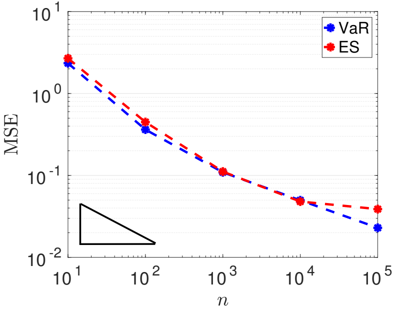
(a) Density Portfolio 1.   (b) Density Portfolio 2.

Figure: Recovered densities of L: ddCOS vs. COS.

Applications of the ddCOS - Risk measures



(a) Portfolio 1: $q = 99\%$.

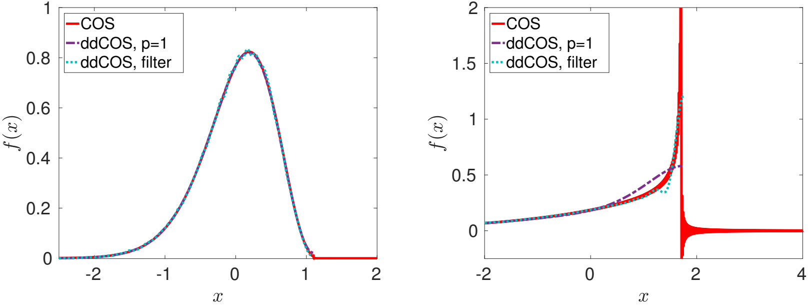


(b) Portfolio 2: $q = 90\%$.

Figure: VaR and ES convergence in n.

Applications of the ddCOS - Risk measures

- The oscillations can be removed.
- Two options: smoothing parameter or filters [RVO14].

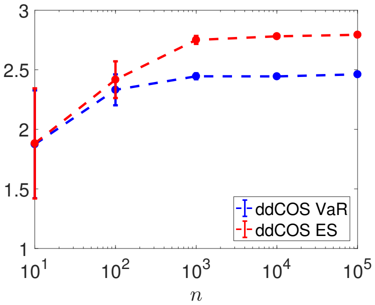


(a) Density Portfolio 1.

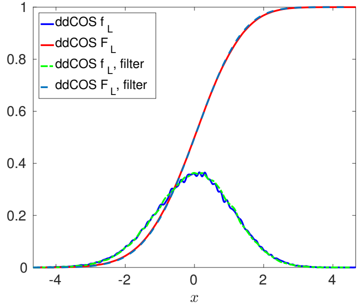
(b) Density Portfolio 2.

Figure: Smoothed densities of L.

Applications of the ddCOS - Risk measures and SABR



(a) VaR and ES: $q = 99\%$.



(b) F_L and f_L.

Figure: Delta-Gamma approach under the SABR model. Setting: $S(0) = 100$, $K = 100$, $r = 0.0$, $\sigma_0 = 0.4$, $\alpha = 0.8$, $\beta = 1.0$, $\rho = -0.5$, $T = 2$, $q = 99\%$ and $\Delta t = 1/365$.

Applications of the ddCOS - Risk measures and SABR

q	10%	30%	50%	70%	90%
VaR	-1.4742	-0.5917	-0.0022	0.5789	1.3862
ES	0.1972	0.5345	0.8644	1.2517	1.8744

Table: VaR and ES under SABR model. Setting: $S(0) = 100$, $K = 100$, $r = 0.0$, $\sigma_0 = 0.4$, $\alpha = 0.8$, $\beta = 1.0$, $\rho = -0.5$, $T = 2$, and $\Delta t = 1/365$.

Conclusions

- The ddCOS method extends the COS method applicability to cases when only data samples of the underlying are available.
- The method exploits a closed-form solution, in terms of Fourier cosine expansions, of a regularization problem.
- It allows to develop a data-driven method which can be employed for option pricing and risk management.
- The ddCOS method particularly results in an efficient method for the Δ and Γ sensitivities computation, based solely on the samples.
- It can be employed within the Delta-Gamma approximation for calculating risk measures.
- A possible future extension may be the use of other basis functions. Haar wavelets are for example interesting since they provide positive densities and allow an efficient treatment of dynamic data.

References

Fang Fang and Cornelis W. Oosterlee.
A novel pricing method for European options based on Fourier-cosine series expansions.
SIAM Journal on Scientific Computing, 31:826–848, 2008.

Fang Fang and Cornelis W. Oosterlee.
Pricing early-exercise and discrete barrier options by Fourier-cosine series expansions.
Numerische Mathematik, 114(1):27–62, 2009.

Luis Ortiz-Gracia and Cornelis W. Oosterlee.
Efficient VaR and Expected Shortfall computations for nonlinear portfolios within the delta-gamma approach.
Applied Mathematics and Computation, 244:16–31, 2014.

Maria J. Ruijter, Mark Versteegh, and Cornelis W. Oosterlee.
On the application of spectral filters in a Fourier option pricing technique.
Journal of Computational Finance, 19(1):75–106, 2014.

Vladimir N. Vapnik.
Statistical Learning Theory.
Wiley-Interscience, 1998.

Suggestions, comments & questions

Thank you for your attention