Continuous Time Markov Chain approximation of the Heston model

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Motivation

- The Heston model is a widely utilized stochastic volatility (SV) models in the option pricing literature as well as in practice.
- For a fixed time horizon, the characteristic function (ChF) is known in closed-form.
- Then, European option pricing is efficiently accomplished with any standard Fourier method.
- Enabling a fast calibration of the Heston model parameters to match observed volatility surfaces, as required in practice.
- However, after calibration there is still great difficulty in pricing exotic contracts under the Heston model.
- To price contracts such as Asian options and variance swaps, Monte Carlo (MC) methods are the traditional surrogates in these cases.
- Unfortunately, MC suffers from a number of well known deficiencies, and complicated simulation schemes are often required to overcome the boundary effects that accompany models such as Heston.

What we propose

- The practical objective of this work is to formalize a model which reproduces vanilla market quotes, but is at the same time amenable to complex derivative pricing in a manner that is consistent with the calibrated model.
- We propose a model and framework based on the Heston model. We call this the CTMC-Heston model, as it uses a finite state *Continuous Time Markov Chain* (CTMC) approximation to the variance process.
- The new formulation enables a closed-form solution for the ChF of the underlying (log-)returns, which allows the use of Fourier inversion techniques to efficiently price exotics.
- We provide numerical studies which demonstrate convergence to Heston's model as the state space is refined. A detailed theoretical analysis of the method will follow.

Outline

1. From Heston model to CTMC-Heston model
2. Calibration of the CTMC-Heston model
3. Application: Pricing Exotic options under CTMC-Heston model
4. Conclusions

From Heston model to CTMC-Heston model

- The Heston stochastic volatility model,

$$\begin{aligned} \frac{\mathrm{d}S_t}{S_t} &= (r-q)\mathrm{d}t + \sqrt{v_t}\mathrm{d}W_t^1, \\ \mathrm{d}v_t &= \eta(\theta - v_t)dt + \sigma_v\sqrt{v_t}\mathrm{d}W_t^2, \end{aligned} \tag{1}$$

  where $\mathrm{d}W_t^1$ and $\mathrm{d}W_t^2$ are correlated Brownian motions, i.e. $\mathrm{d}W_t^1\mathrm{d}W_t^2 = \rho\mathrm{d}t$, with $\rho \in (-1, 1)$.
- The stochastic volatility (or variance), v_t, is driven by a *CIR* process, having a mean reversion component.
- Value v_0 is the initial volatility, η controls the mean reversion speed while θ is the long-term volatility and σ_v corresponds to the volatility of the variance process v_t, also known as *vol-vol* (volatility-of-volatility).
- The model parameters are therefore $\Theta = \{v_0, \eta, \theta, \sigma_v, \rho\}$.

- The Heston's model solution can be re-expressed in the form

$$\log\left(\frac{S_t}{S_0}\right) = \frac{\rho}{\sigma_v}(v_t - v_0) + (r-q)t - \frac{1}{2}\int_0^t v_s \mathrm{d}s$$
$$-\frac{\rho}{\sigma_v}\int_0^t \eta(\theta - v_s)\mathrm{d}s + \sqrt{1-\rho^2}\int_0^t \sqrt{v_s}\mathrm{d}W_s^*,$$

where $W_t^1 := \rho W_t^2 + \sqrt{1-\rho^2}W_t^*$ and W_t^* is independent from W_t^2.

- Rearranging, we introduce the auxiliary process $(\widetilde{X}_t)_{t\geq 0}$,

$$\widetilde{X}_t := \log\left(\frac{S_t}{S_0}\right) - \frac{\rho}{\sigma_v}(v_t - v_0)$$
$$= \left(r - q - \frac{\rho\eta\theta}{\sigma_v}\right)t + \left(\frac{\rho\eta}{\sigma_v} - \frac{1}{2}\right)\int_0^t v_s \mathrm{d}s + \sqrt{1-\rho^2}\int_0^t \sqrt{v_s}\mathrm{d}W_s^*.$$

- We thus have the following uncoupled two-factor representation,

$$\mathrm{d}\widetilde{X}_t = \left[\left(\frac{\rho\eta}{\sigma_v} - \frac{1}{2}\right)v_t + \bar{\omega}\right]\mathrm{d}t + \sqrt{(1-\rho^2)v_t}\mathrm{d}W_t^*,$$
$$\mathrm{d}v_t = \mu(v_t)\mathrm{d}t + \sigma(v_t)\mathrm{d}W_t^2,$$

where $\bar{\omega} := \left(r - q - \frac{\rho\eta\theta}{\sigma_v}\right)$, $\mu(v_t) := \eta(\theta - v_t)$ and $\sigma(v_t) := \sigma_v\sqrt{v_t}$.

CTMC-Heston model

- Given a state-space $\mathbf{v} := \{v_1, \ldots, v_{m_0}\}$, and a CTMC $\{\alpha(t), t \geq 0\}$ transitioning between the indexes $\{1, \ldots, m_0\}$ according to

$$\mathbb{Q}\{\alpha(t+\Delta t) = j | \alpha(t) = k\} = \delta_{jk} + q_{jk}\Delta t + o(\Delta t).$$

- The set of transition rates q_{jk} form the *generator* matrix $Q_{m_0 \times m_0}$, chosen so that $(v_{\alpha(t)})_{t\geq 0}$ are locally consistent with $(v_t)_{t\geq 0}$.
- Given $(v_{\alpha(t)})_{t\geq 0}$, $\widetilde{X}_t$ is approximated by a Regime Switching (RS) diffusion,

$$\begin{aligned}\widetilde{X}_t^{\alpha} &= \bar{\omega} t + \int_0^t \left(\frac{\rho\eta}{\sigma_v} - \frac{1}{2}\right) v_{\alpha(s)} ds + \sqrt{1-\rho^2} \int_0^t \sqrt{v_{\alpha(s)}} dW^*(s) \\ &= \int_0^t \zeta_{\alpha(s)} ds + \int_0^t \beta_{\alpha(s)} dW^*(s),\end{aligned}$$

where for $\alpha(s) \in \{1, \ldots, m_0\}$,

$$\zeta_{\alpha(s)} := \left(r - q - \frac{\rho\eta\theta}{\sigma_v}\right) + \left(\frac{\rho\eta}{\sigma_v} - \frac{1}{2}\right) v_{\alpha(s)}, \quad \beta_{\alpha(s)} := \sqrt{(1-\rho^2) v_{\alpha(s)}}.$$

- Main advantage: the new formulation enables a closed-form expression for the conditional ChF. Given $\Delta t > 0$, $\forall j = 1, ..., m_0$,

$$\widetilde{\phi}^{j}_{\widetilde{X}^{\alpha}_{\Delta t}}(\xi) := \mathbb{E}[e^{\mathrm{i}\xi \widetilde{X}^{\alpha}_{\Delta t}} | \alpha(0 \le s \le \Delta t) = j]$$
$$= \mathbb{E}\left[\exp\left(\mathrm{i}\xi\left(\zeta_j \Delta t + \beta_j W^*(\Delta t)\right)\right)\right] := \exp(\psi_j(\xi)\Delta t),$$

where $\psi_j(\xi) = \mathrm{i}\zeta_j\xi - \frac{1}{2}\xi^2\beta_j^2, j = 1, \ldots, m_0$. is its *Lévy symbol*.
- The process $\widetilde{X}^{\alpha}_{t}$ is completely characterized by the set $\{\psi_j(\xi)\}_{j=1}^{m_0}$, together with the generator Q.
- The ChF of $\widetilde{X}^{\alpha}_{\Delta t}$, $\Delta t \ge 0$, conditioned on the initial state $\alpha(0) = j_0$,

$$\mathbb{E}\left[e^{\mathrm{i}\widetilde{X}^{\alpha}_{\Delta t}\xi} | \alpha(0) = j_0\right] = \mathbf{1}'\mathcal{M}(\xi; \Delta t)\mathbf{e}_{j_0}, \quad j_0 \in \{1, \ldots, m_0\}$$

where we define the matrix exponential

$$\mathcal{M}(\xi; \Delta t) := \exp\left(\Delta t\left(Q' + \mathrm{diag}(\psi_1(\xi), \ldots, \psi_{m_0}(\xi))\right)\right),$$

and $\mathbf{1} \in \mathbb{R}^{m_0}$ represents a column vector of ones, and $\mathbf{e}_j \in \mathbb{R}^{m_0}$ a unit column vector with a one in position j.

- $\widetilde{X}^{\alpha}_{\Delta t}$ induces the following *CTMC-Heston model* for the underlying $S_{\Delta t}$, namely

$$S^{\alpha}_{\Delta t} = S_0 \exp\left(\widetilde{X}^{\alpha}_{\Delta t} + \frac{\rho}{\sigma_v}(v_{\alpha(\Delta t)} - v_{\alpha(0)})\right).$$

- The conditional ChF of the log-increment

$$R^{\alpha}_{\Delta t} := \log(S^{\alpha}_{\Delta t}/S_0) = \widetilde{X}^{\alpha}_{\Delta t} + \frac{\rho}{\sigma_v}(v_{\alpha(\Delta t)} - v_{\alpha(0)}),$$

is recovered in closed-form as

$$\begin{aligned}\mathbb{E}[e^{\mathrm{i}R^{\alpha}_{\Delta t}\xi}|\alpha(0) = j, \alpha(\Delta t) = k] &= \mathcal{M}_{k,j}(\xi;\Delta t)\cdot\exp\left(\mathrm{i}\xi\frac{\rho}{\sigma_v}(v_k - v_j)\right)\\ &:= \widetilde{\mathcal{M}}_{k,j}(\xi;\Delta t).\end{aligned}$$

which follows from conditional independence.

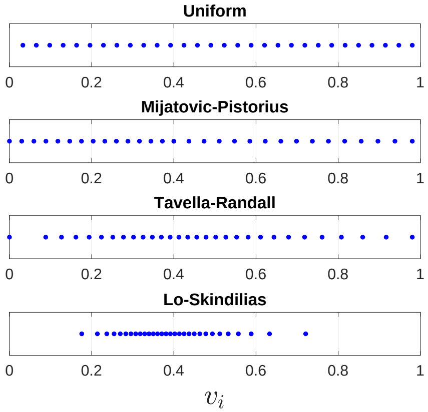
- We can view the CTMC-Heston model as both an approximation to Heston's model, as well as a tractable model in its own right.

Calibration of the CTMC-Heston model

- As a Fourier inversion method we employ *SWIFT*, which has several important advantages which make it well-suited for calibration:
  - **Error control**. It is probably the most relevant property within an optimization problem. Thanks to the use of Shannon wavelets, SWIFT establishes a bound in the error given any scale m of approximation.
  - **Robustness**. SWIFT provides mechanisms to determine all the free parameters in the approximation made based on the scale m which, as mentioned in the previous point, determines the committed error.
  - **Performance efficiency**. As other Fourier inversion techniques, SWIFT is an extremely fast algorithm, allowing FFT, vectorized operations or even parallel computing features.
  - **Accuracy**. Although an error bound is provided, SWIFT has demonstrated a very high precision in most situations, far below the predicted error bound and, at least, comparable with the state-of-the-art methodologies.
- The properties mentioned above ensure high quality estimations in the calibration process, reducing the chances of any possible malfunctioning or divergence in the optimization procedure.

Grid selection

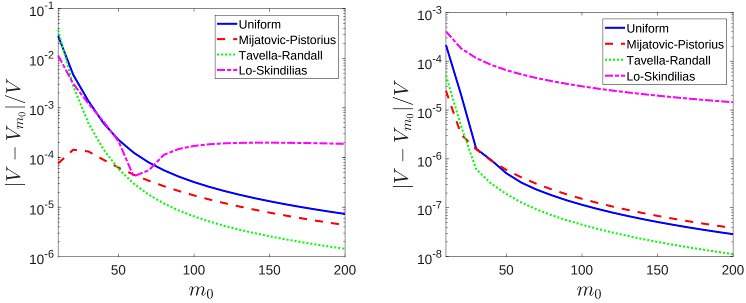
- Our goal is to form a model which parsimoniously resembles Heston.
- One of the key aspects in designing the CTMC-Heston model is a specification for the variance state-space (grid).
- Several conceptually different approaches available in the literature.



- Data sets: two representative scenarios.

	scenario	v_0	η	θ	σ_v	ρ
Set I	regular market	0.03	3.0	0.04	0.25	-0.7
Set II	stressed market	0.4	3.0	0.4	0.5	-0.1

- Convergence in m_0.



(a) Set I (b) Set II

Influence of the model parameters

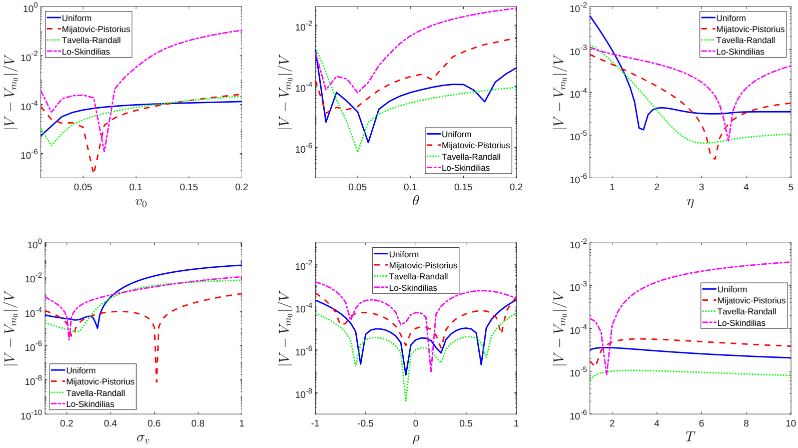


Figure: Set I: put option, $S_0 = 100$, $K = 100$, $r = 0.05$ and $T = 1$.

Influence of the model parameters

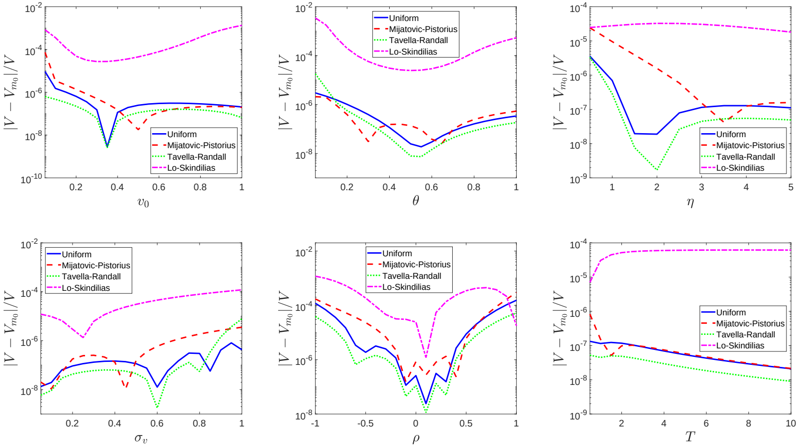
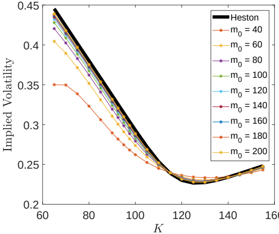
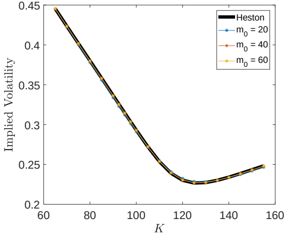


Figure: Set II: put option, $S_0 = 100$, $K = 100$, $r = 0.05$ and $T = 1$.

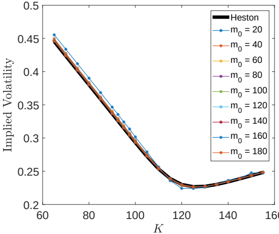
Calibration with real data (Microsoft, January 2019)



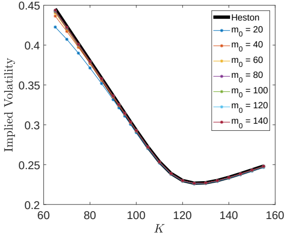
(a) Uniform.



(b) Mijatovic-Pistorius

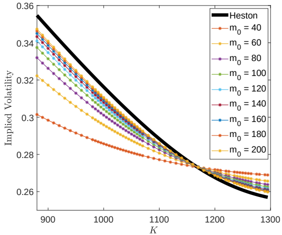


(c) Tavella-Randall

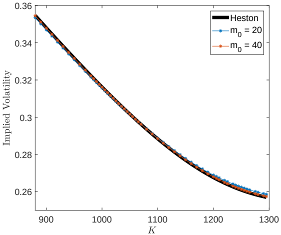


(d) Lo-Skindilias

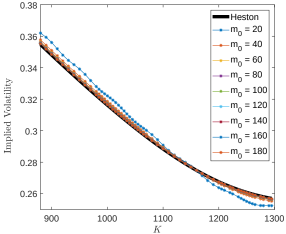
Calibration with real data (Google, January 2019)



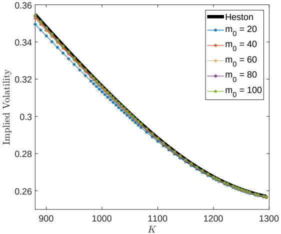
(a) Uniform.



(b) Mijatovic-Pistorius



(c) Tavella-Randall



(d) Lo-Skindilias

Interesting lessons

- All the approaches provide numerical convergence in m_0.
- The error decays very fast at the beginning, with smaller m_0, and smoothens for bigger m_0, suggesting a damping effect.
- The grid distribution proposed by Lo and Skindilias provides, in general, poorer estimations.
- Although the uniform approach performs surprisingly well in the test with synthetic parameters, the real calibration experiment shows a pretty inaccurate estimations for options far from at-the-money strike.
- The schemes by Mijatovic-Pistorius and Tavella-Randall perform similarly. It is worth noting that the first explodes when the initial and long-term volatilities differ greatly one from the other. The second happens to be the most robust and precise choice in general.
- By focusing on the correlation parameter, ρ, in the second test, we observe that the error tends to be minimum close to the no-correlation point ($\rho = 0$), and it degrades when ρ ventures far form zero.

Application: Exotic options under CTMC-Heston model

- Once calibrated, a model is commonly employed to price more involved products (early-exercise, path-dependent, etc.).
- Many exotic products can be defined in terms of a generic recursion.
- Consider $N+1$ monitoring dates, $0 = t_0 < t_1 < \cdots < t_N = T$. We define the log returns R_n by

$$R_n := \log\left(\frac{S_n}{S_{n-1}}\right), \quad S_n := S(t_n), \quad n = 1, ..., N.$$

- The contracts of interest satisfy a very general sequence of equations

$$Y_1 := w_N \cdot h(R_N) + \varrho_N$$
$$Y_n := w_{N-(n-1)} \cdot h(R_{N-(n-1)}) + g(Y_{n-1}) + \varrho_{N-(n-1)}, \quad n = 2, \ldots, N,$$

where h, g are continuous functions, $\{w_n\}_{n=1}^N$ is a set of weights, and $\{\varrho_n\}_{n=1}^N$ is a set of shift parameters. Includes contracts of the form

$$G\left(\sum_{n=1}^{N} w_n \cdot h(R_n); \Theta\right).$$

- Prominent examples of contracts which fall within this framework.
  - **Realized variance swaps and options:**

$$A_N = \frac{1}{T}\sum_{n=1}^{N}(R_n)^2 \quad \text{and} \quad A_N = \frac{1}{T}\sum_{n=1}^{N}(\exp(R_n)-1)^2,$$

    with $G(A_N) := A_N - K$ (swap), and $G(A_N) := (A_N - K)^+$ (call).
  - **Cliquets:** with local (global) floor and cap F, G (F_g, G_g),

$$A_N = \sum_{n=1}^{N}\max\left(F, \min\left(C, \exp(R_n)-1\right)\right),$$

    with $G(A_N) = K \cdot \min\left(C_g, \max\left(F_g, A_N\right)\right)$.
  - **Arithmetic (weighted) Asian Options:**

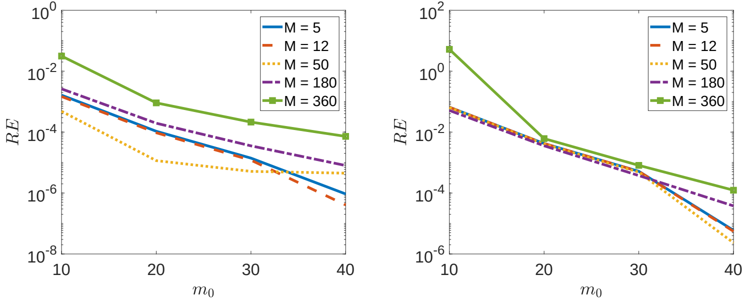
$$\begin{aligned} A_N &:= \frac{1}{N+1}\sum_{n=0}^{N} w_n S_n \\ &= \frac{S_0}{N+1}\left(w_0 + e^{R_1}\left(w_1 + e^{R_2}\left(\cdots e^{R_{N-1}}\left(w_{N-1} + w_N e^{R_N}\right)\right)\right)\right), \end{aligned}$$

    where $G(A_N) := (A_N - K)^+$ for a call option.

Numerical experiments with exotic options

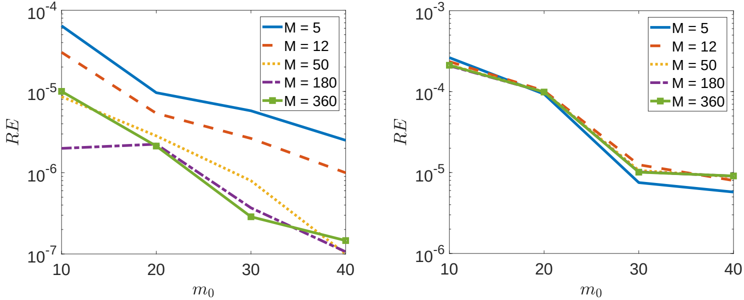
- We will present some experiments aiming to numerically validate the introduced CTMC-Heston model.
- We will consider several exotic contracts: realized variance swaps, realized variance options and Asian options.
- The recursive definition above allows efficient Fourier methods (SWIFT).
- The realized variance swaps are chosen for comparative purposes, since an exact solution for the Heston model is available.
- That is not the case for the other two products, which often require the use of MC methods.
- Computer system CPU Intel Core i7-4720HQ 2.6GHz, 16GB RAM and Matlab R2017b.
- Based on the calibration tests, Tavella-Randall scheme is used.
- MC setting: QE scheme with 10^6 paths and 360 time steps.

Convergence in m_0



(a) $\rho = -0.1$. (b) $\rho = -0.7$.

Figure: Variance Swaps: $r = 0.05$ and $T = 1$. Heston parameters: Set I (regular market). Grid Design: Tavella-Randall.



(a) $\rho = -0.1$. (b) $\rho = -0.7$.

Figure: Variance Swaps: $r = 0.05$ and $T = 1$. Heston parameters: Set II (stressed market). Grid Design: Tavella-Randall.

Realized variance swaps (Set I)

	$\rho = -0.1$				
N	Ref.	SWIFT	MC	RE_{SWIFT}	RE_{MC}
5	0.0371205474	0.0371205820	0.0371242281	9.32×10^{-7}	9.91×10^{-5}
12	0.0369570905	0.0369571055	0.0369627242	4.06×10^{-7}	1.52×10^{-4}
50	0.0368631686	0.0368630034	0.0368736021	4.48×10^{-6}	2.83×10^{-4}
180	0.0368411536	0.0368414484	0.0368357466	8.00×10^{-6}	1.46×10^{-4}
360	0.0368368930	0.0368342265	0.0368466261	7.23×10^{-5}	2.64×10^{-4}
	$\rho = -0.7$				
N	Ref.	SWIFT	MC	RE_{SWIFT}	RE_{MC}
5	0.0375737983	0.0375740073	0.0375539243	5.56×10^{-6}	5.28×10^{-4}
12	0.0371685246	0.0371687309	0.0371532126	5.55×10^{-6}	4.11×10^{-4}
50	0.0369172829	0.0369172021	0.0369199786	2.18×10^{-6}	7.30×10^{-5}
180	0.0368564120	0.0368549997	0.0368514021	3.83×10^{-5}	1.35×10^{-4}
360	0.0368445443	0.0368489009	0.0368522457	1.18×10^{-4}	2.09×10^{-4}

Table: Variance Swaps: $m_0 = 40$, Set I, $T = 1$ $r = 0.05$.

Realized variance swaps (Set II)

	$\rho = -0.1$				
N	Ref.	SWIFT	MC	RE_{SWIFT}	RE_{MC}
5	0.4067078727	0.4067086532	0.4065423061	1.91×10^{-6}	4.07×10^{-4}
12	0.4029056015	0.4029060040	0.4027957415	9.99×10^{-7}	2.72×10^{-4}
50	0.4007139003	0.4007139406	0.4008416719	1.00×10^{-7}	3.18×10^{-4}
180	0.4001994199	0.4001993773	0.4002238554	1.06×10^{-7}	6.10×10^{-5}
360	0.4000998185	0.4000997601	0.4000181976	1.46×10^{-7}	2.04×10^{-4}
	$\rho = -0.7$				
N	Ref.	SWIFT	MC	RE_{SWIFT}	RE_{MC}
5	0.4166286485	0.416631041793736	0.4167251660	5.74×10^{-6}	2.31×10^{-4}
12	0.4075137267	0.4075104743	0.4073002992	7.98×10^{-6}	5.23×10^{-4}
50	0.4018902561	0.4018867030	0.4019702179	8.84×10^{-6}	1.98×10^{-4}
180	0.4005309091	0.4005272887	0.4006611714	9.03×10^{-6}	3.25×10^{-4}
360	0.4002660232	0.4002623900	0.4001430561	9.07×10^{-6}	3.07×10^{-4}

Table: Variance Swaps: $m_0 = 40$, Set II, $T = 1$ $r = 0.05$.

Realized variance option

	$\rho = -0.1$			$\rho = -0.7$		
K	Ref.(MC)	SWIFT	RE	Ref.(MC)	SWIFT	RE
0.01	0.02567765	0.02568006	9.40×10^{-5}	0.02587552	0.02586686	3.34×10^{-4}
0.02	0.01699106	0.01701443	1.37×10^{-3}	0.01712666	0.01710650	1.17×10^{-3}
0.03	0.01045427	0.01044283	1.09×10^{-3}	0.01053466	0.01054260	7.57×10^{-4}
0.04	0.00613621	0.00613145	7.75×10^{-4}	0.00631007	0.00633681	4.23×10^{-3}
0.05	0.00351388	0.00352261	2.48×10^{-3}	0.00380057	0.00380673	1.62×10^{-3}

Table: Variance Call Options: $m_0 = 40$, $T = 1$, $r = 0.05$, $N = 12$. Heston Set I.

	$\rho = -0.1$			$\rho = -0.7$		
K	Ref.(MC)	SWIFT	RE	Ref.(MC)	SWIFT	RE
0.1	0.28810430	0.28826035	5.41×10^{-4}	0.29269761	0.29262732	2.40×10^{-4}
0.2	0.19753250	0.19753794	2.75×10^{-5}	0.20203744	0.20184267	9.64×10^{-4}
0.3	0.12269943	0.12276166	5.07×10^{-4}	0.12730330	0.12737500	5.63×10^{-4}
0.4	0.07050097	0.07054838	6.72×10^{-4}	0.07568155	0.07567259	1.18×10^{-4}
0.5	0.03826162	0.03836334	2.65×10^{-3}	0.04341057	0.04352151	2.55×10^{-3}

Table: Variance Call Options: $m_0 = 40$, $T = 1$, $r = 0.05$, $N = 12$. Heston Set II.

Arithmetic Asian option (Set I)

$N = 12$			
$K(\% of S_0)$	Ref.(MC)	SWIFT	RE
80%	21.5285835237	21.5270366207	7.18×10^{-5}
90%	12.5823808044	12.5896547750	5.78×10^{-4}
100%	5.4002621022	5.4000546644	3.84×10^{-5}
110%	1.3880527793	1.3906598970	1.87×10^{-3}
120%	0.1736330491	0.1731034094	3.05×10^{-3}
	$N = 50$		
$K(\% of S_0)$	Ref.(MC)	SWIFT	RE
80%	21.5386392371	21.5339280578	2.18×10^{-4}
90%	12.6239658563	12.6182127196	4.55×10^{-4}
100%	5.4504220302	5.4499634141	8.41×10^{-5}
110%	1.4295579101	1.4275471949	1.40×10^{-3}
120%	0.1824925012	0.1831473714	3.58×10^{-3}
	$N = 250$		
$K(\% of S_0)$	Ref.(MC)	SWIFT	RE
80%	21.5266346261	21.5359313401	4.31×10^{-4}
90%	12.6269859960	12.6261325064	6.75×10^{-5}
100%	5.4534882341	5.4636939471	1.87×10^{-3}
110%	1.4440819439	1.4378101025	4.34×10^{-3}
120%	0.1875776074	0.1860298190	8.25×10^{-3}

Table: Tavella-Randall, $m_0 = 40$, Set I, call, option, $S_0 = 100$, $T = 1$, $r = 0.05$.

Arithmetic Asian option (Set II)

$N = 12$			
$K(\%ofS_0)$	Ref.(MC)	SWIFT	RE
80%	25.5585678735	25.5988860602	1.57×10^{-3}
90%	19.6670725943	19.6278689575	1.99×10^{-3}
100%	14.8962382700	14.8552716759	2.75×10^{-3}
110%	11.1517895745	11.1463503256	4.87×10^{-4}
120%	8.3165299338	8.3212712111	5.70×10^{-4}
	$N = 50$		
$K(\%ofS_0)$	Ref.(MC)	SWIFT	RE
80%	25.7824036750	25.7778794489	1.75×10^{-4}
90%	19.8263899575	19.8272466858	4.32×10^{-5}
100%	15.0530165896	15.0529723969	2.93×10^{-6}
110%	11.3291439277	11.3270969069	1.80×10^{-4}
120%	8.4614191560	8.4772028373	1.86×10^{-3}
	$N = 250$		
$K(\%ofS_0)$	Ref.(MC)	SWIFT	RE
80%	25.8641465886	25.8255340288	1.49×10^{-3}
90%	19.9219203435	19.8806560654	2.07×10^{-3}
100%	15.1245760541	15.1064333350	1.19×10^{-3}
110%	11.3793305624	11.3765266399	2.46×10^{-4}
120%	8.5254366308	8.5203790223	5.93×10^{-4}

Table: Tavella-Randall, $m_0 = 40$, Set II, call, option, $S_0 = 100$, $T = 1$, $r = 0.05$.

Conclusions

- This work provides a general, computationally efficient, and robust valuation framework under the CTMC-Heston model.
- This model approximation provides a parsimonious and faithful representation of the Heston model, and it is able to reproduce the same volatility smile structure with a modest number of states.
- We can efficiently price a large variety of contracts which are exceptionally difficult to handle under Heston's model.
- The efficiency of the method is obtained by combining the CTMC approximation of the variance, with the SWIFT Fourier method.
- An extensive set of numerical experiments were provided, analyzing Asian options and discretely sampled realized variance derivatives.
- A detailed error analysis will follow (work in progress).

References

Zhenyu Cui, Justin L. Kirkby, and Duy Nguyen.
Springer IMA volume: Recent Developments in Financial and Economic Applications, chapter Continuous-Time Markov Chain and Regime Switching approximations with applications to options pricing.
Forthcoming, Springer, 2019.

Álvaro Leitao, Luis Ortiz-Gracia, and Emma I. Wagner.
SWIFT valuation of discretely monitored arithmetic Asian options.
Journal of Computational Science, 28:120–139, 2018.

Chia Chun Lo and Konstantinos Skindilias.
An improved Markov chain approximation methodology: derivatives pricing and model calibration.
International Journal of Theoretical and Applied Finance, 17(07):1450047, 2014.

Aleksandar Mijatović and Martijn Pistorius.
Continuously monitored barrier options under Markov processes.
Mathematical Finance, 23(1):1–38, 2013.

Domingo Tavella and Curt Randall.
Pricing Financial Instruments: The Finite Difference Method.
Wiley, 2000.

Acknowledgements & Questions

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More: `alvaroleitao.github.io`

Thank you for your attention

- Given a grid of points $\mathbf{v} = \{v_1, v_2, \ldots, v_{m_0}\}$ with grid spacings $h_i = v_{i+1} - v_i$, and assuming that $v_{\alpha(t)}$ takes values on $\mathbf{v}$, the elements q_{ij} of the generator Q for the CTMC approximation of the process v_t read

$$q_{ij} = \begin{cases} \dfrac{\mu^-(v_i)}{h_{i-1}} + \dfrac{\sigma^2(v_i) - (h_{i-1}\mu^-(v_i) + h_i\mu^+(v_i))}{h_{i-1}(h_{i-1}+h_i)}, & \text{if } j = i-1, \\ \dfrac{\mu^+(v_i)}{h_i} + \dfrac{\sigma^2(v_i) - (h_{i-1}\mu^-(v_i) + h_i\mu^+(v_i))}{h_i(h_{i-1}+h_i)}, & \text{if } j = i+1, \\ -q_{i,i-1} - q_{i,i+1}, & \text{if } j = i, \\ 0, & \text{otherwise}, \end{cases}$$

with the notation $z^\pm = \max(\pm z, 0)$. Further, to guarantee a well-defined probability matrix, the following condition must be satisfied:

$$\max_{1 \le i < m_0} (h_i) \le \min_{1 \le i \le m_0} \left(\frac{\sigma^2(v_i)}{|\mu(v_i)|} \right).$$