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A stochastic θ-SEIHRD model

Adding randomness to the COVID-19 spread

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Motivation

- Classical compartmental SEIR-like models are too simplistic.
- Particular COVID-19 characteristics: undetected, hospitalized, deaths, etc.
- Require a COVID-19 *ad-hoc* model: θ-SEIHRD model.
- Deterministic version: rigid and limited information.
- Uncertainty may influence the compartments dynamics.
- Behavioural effects, public interventions, seasonal patterns, environmental factors, etc. are factors with a random component.
- How to account for it? Stochastic extension!

Outline

- The compartmental models are formulated in a deterministic fashion: *Ordinary Differential Equations* (ODEs).
- There are two common approaches to include stochasticity into a deterministic model:
  - *Continuous Time Markov Chain* (CTMC).
  - *Stochastic Differential Equations* (SDEs).
- The stochastic models allow to capture many kinds of circumstances including uncertainty.
- The solution of the stochastic model is a set of stochastic processes, containing much more information than the deterministic analogous.
- Statistical analyses can be performed (expectations, quantiles or worst case scenarios).

- We follow the SDE approach incorporating a *Brownian motion* to the ODEs.
- Two common ways of addressing this kind of stochastic extension:
  - Adding arbitrary random noise.
  - Perturbing one (o more) of the existing model parameters.
- We choose the second alternative for interpretability purposes.
- In practice, the uncertainty will have impact on a particular model component, typically represented by a model parameter.
- A randomly perturbed parameter can be reasonably explained in terms of the variability produced by the source of the considered uncertainty.

- We consider the (simplified) θ-SEIHRD model from [2].
- Consisting in 9 equations, 6 coupled,

$$\begin{aligned}
\frac{\mathrm{d}S}{\mathrm{d}t}(t) &= -\frac{S(t)}{N}\left(m_E(t)\beta_E E(t) + m_I(t)\beta_I I(t) + m_{I_u}(t)\beta_{I_u}(\theta(t))I_u(t)\right) \\
&\quad - \frac{S(t)}{N}\left(m_{H_R}(t)\beta_{H_R}(t)H_R(t) + m_{H_D}(t)\beta_{H_D}(t)H_D(t)\right), \\
\frac{\mathrm{d}E}{\mathrm{d}t}(t) &= \frac{S(t)}{N}\left(m_E(t)\beta_E E(t) + m_I(t)\beta_I I(t) + m_{I_u}(t)\beta_{I_u}(\theta(t))I_u(t)\right) \\
&\quad + \frac{S(t)}{N}\left(m_{H_R}(t)\beta_{H_R}(t)H_R(t) + m_{H_D}(t)\beta_{H_D}(t)H_D(t)\right) - \gamma_E E(t), \\
\frac{\mathrm{d}I}{\mathrm{d}t}(t) &= \gamma_E E(t) - \gamma_I(t)I(t), \\
\frac{\mathrm{d}I_u}{\mathrm{d}t}(t) &= (1-\theta(t))\gamma_I(t)I(t) - \gamma_{I_u}(t)I_u(t), \\
\frac{\mathrm{d}H_R}{\mathrm{d}t}(t) &= \theta(t)\left(1 - \frac{\omega(t)}{\theta(t)}\right)\gamma_I(t)I(t) - \gamma_{H_R}(t)H_R(t), \\
\frac{\mathrm{d}H_D}{\mathrm{d}t}(t) &= \omega(t)\gamma_I(t)I(t) - \gamma_{H_D}(t)H_D(t),
\end{aligned}$$

- and 3 uncoupled equations,

$$\frac{\mathrm{d}R_d}{\mathrm{d}t}(t) = \gamma_{H_R}(t)H_R(t),$$
$$\frac{\mathrm{d}R_u}{\mathrm{d}t}(t) = \gamma_{I_u}(t)I_u(t),$$
$$\frac{\mathrm{d}D}{\mathrm{d}t}(t) = \gamma_{H_D}(t)H_D(t).$$

- whose solution can be obtained by

$$R_d(t) = R_d(t_0) + \int_{t_0}^{t} \gamma_{H_R}(s)H_R(s)\mathrm{d}s,$$
$$R_u(t) = R_u(t_0) + \int_{t_0}^{t} \gamma_{I_u}(s)I_u(s)\mathrm{d}s,$$
$$D(t) = D(t_0) + \int_{t_0}^{t} \gamma_{H_D}(s)H_D(s)\mathrm{d}s,$$

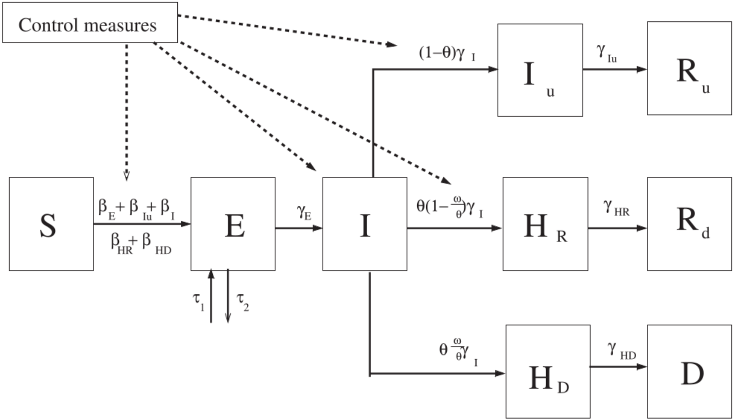


Figure: The θ-SEIHRD model diagram.

- Efficiency of the control measures $m_E, m_I, m_{I_u}, m_{H_R}, m_{H_D} \in [0,1](\%)$. Here, only one control measure is assumed, implemented at date λ_1,

$$m_X(t) = \begin{cases} 1, & \text{if } t \in [0, \lambda_1], \\ \exp\left(-\kappa_1(t - \lambda_1)\right), & \text{if } t \in [\lambda_1, T], \end{cases}$$

  with the parameter $\kappa_1 \in [0, 0.2]$.

- The fatality rate $\omega(t) \in [\underline{\omega}, \overline{\omega}] \subset [0,1]$,

$$\omega(t) = m_I(t)\overline{\omega} + (1 - m_I(t))\underline{\omega},$$

  with $\underline{\omega}$ and $\overline{\omega}$ being the fatality rate limits with and without control measures, $\overline{\omega} = \underline{\omega} + \delta_\omega$.

- The fraction of detected individuals, $\theta \in [\overline{\omega}, 1]$,

$$\theta(t) = \begin{cases} \underline{\theta}, & \text{if } t \in [t, \lambda_1], \\ \text{linear continuous}, & \text{if } t \in [\lambda_1, \lambda_2], \\ \overline{\theta}, & \text{if } t \in [\lambda_2, T], \end{cases}$$

with $\underline{\theta}, \overline{\theta}, \lambda_1, \lambda_2$ inferred from the data.

- Compartment transition rates $\gamma_E, \gamma_I, \gamma_{I_u}, \gamma_{H_R}, \gamma_{H_D} \in (0, +\infty)$. Given the days in each compartment, d_E, d_I, d_{I_u}, d_{H_R} and d_{H_D}, with $d_{I_u} = d_{H_R}$ and $d_{H_D} = d_{H_R} + \delta_R, \delta_R > 0$,

$$\gamma_I = \frac{1}{d_E}, \qquad \gamma_{I_u}(t) = \gamma_{H_R}(t) = \frac{1}{d_{I_u} + g(t)}$$

$$\gamma_I(t) = \frac{1}{d_I - g(t)}, \qquad \gamma_{H_D}(t) = \frac{1}{d_{I_u} + g(t) + \delta_R},$$

where $g(t) = d_g(1 - m_I(t))$.

- The disease contact rates $\beta_E, \beta_I, \beta_{I_u}, \beta_{H_R}, \beta_{H_D} \in \mathbb{R}^+$. The parameter β_I is calibrated. More $\beta_E = C_E\beta_I$, $\beta_{H_R} = \beta_{H_D} = C_H(t)\beta_I$ and

$$\beta_{I_u}(t) = \underline{\beta}_I + \frac{\beta_I - \underline{\beta}_I}{1-\omega(t)}(1-\theta(t)),$$

where $\underline{\beta}_I = C_u\beta_I$, with C_E, $C_H(t)$ and $C_u \in [0,1]$. Parameters C_E and C_u are obtained calibration, while

$$C_H(t) = \frac{\alpha_H\left(\frac{\beta_I}{\gamma_I(t)} + \frac{\beta_E}{\gamma_E(t)} + (1-\theta(t))\frac{\beta_{I_u}(t)}{\gamma_{I_u}(t)}\right)}{(1-\alpha_H)\beta_I\theta(t)\left(\left(1-\frac{\omega(t)}{\theta(t)}\right)\frac{1}{\gamma_{H_R}(t)} + \frac{\omega(t)}{\theta(t)}\frac{1}{\gamma_{H_R}(t)}\right)}$$

with α_H being the percentage of healthcare workers infected.

- Our aim is to introduce stochasticity to the simplified θ-SEIHRD model.
- We add randomness on the disease contact rates, β's.
- Writing them in terms of β_I,

$$\beta_E = \beta_I A_E, \quad \beta_{I_u} = \beta_I A_{I_u}, \quad \beta_{H_R} = \beta_I A_{H_R}, \quad \beta_{H_D} = \beta_I A_{H_D},$$

where

$$\begin{aligned} A_E &= C_E, \\ A_{I_u}(t) &= C_u + \frac{(1-C_u)(1-\theta(t))}{1-\omega(t)}, \\ A_{H_R}(t) = A_{H_D}(t) &= \frac{\alpha_H \left(\frac{1}{\gamma_I(t)} + \frac{A_E}{\gamma_E} + (1-\theta(t)) \frac{A_{I_u}(t)}{\gamma_{I_u}(t)} \right)}{(1-\alpha_H)\theta(t) \left(\left(1 - \frac{\omega(t)}{\theta(t)}\right) \frac{1}{\gamma_{H_R}(t)} + \frac{\omega(t)}{\theta(t)} \frac{1}{\gamma_{H_D}(t)} \right)}. \end{aligned}$$

- The θ-SEIHRD model can be rewritten as,

$$
\begin{aligned}
\frac{\mathrm{d}S}{\mathrm{d}t}(t) &= -\beta_I \frac{S(t)M(t)}{N}, \\
\frac{\mathrm{d}E}{\mathrm{d}t}(t) &= \beta_I \frac{S(t)M(t)}{N} - \gamma_E E(t), \\
\frac{\mathrm{d}I}{\mathrm{d}t}(t) &= \gamma_E E(t) - \gamma_I I(t), \\
\frac{\mathrm{d}I_u}{\mathrm{d}t}(t) &= (1-\theta(t))\gamma_I I(t) - \gamma_{I_u} I_u(t), \\
\frac{\mathrm{d}H_R}{\mathrm{d}t}(t) &= \theta(t)\left(1 - \frac{\omega(t)}{\theta(t)}\right)\gamma_I I(t) - \gamma_{H_R} H_R(t), \\
\frac{\mathrm{d}H_D}{\mathrm{d}t}(t) &= \omega(t)\gamma_I I(t) - \gamma_{H_D} H_D(t),
\end{aligned}
$$

where

$$
\begin{aligned}
M(t) = {} & m_E A_E E(t) + m_I I(t) + m_{I_u} A_{I_u} I_u(t) \\
& + m_{H_R} A_{H_R} H_R(t) + m_{H_D} A_{H_D} H_D(t).
\end{aligned}
$$

- Replace the constant parameter β_I by a random walk.
- The disease contact rate in compartment I follows a newly introduced stochastic process, $\tilde{\beta}_I(t)$.
- We choose the well-known CIR process [1].
- The main advantage of the CIR process: it ensures the spacial states to be non-negative.
- Further, the CIR process is a mean-reverting process.
- The dynamics of $\tilde{\beta}_I$ read

$$\mathrm{d}\tilde{\beta}_I(t) = \nu_{\beta_I}(\mu_{\beta_I} - \tilde{\beta}_I(t))\mathrm{d}t + \sigma_{\beta_I}\sqrt{\tilde{\beta}(t)}\mathrm{d}W(t)$$

  where ν_{β_I} is the mean reverting speed, μ_{β_I} is the long-term average, σ_{β_I} is the volatility and $\mathrm{d}W(t)$ is a Brownian motion increment.

- The system of SDEs governing the stochastic θ-SEIHRD model is given by,

$$
\begin{aligned}
\mathrm{d}S(t) &= \tilde{\beta}_I(t)\frac{S(t)M(t)}{N}\mathrm{d}t\\
\mathrm{d}E(t) &= \left(\tilde{\beta}_I(t)\frac{S(t)M(t)}{N} - \gamma_E E(t)\right)\mathrm{d}t\\
\mathrm{d}I(t) &= \left(\gamma_E E(t) - \gamma_I I(t)\right)\mathrm{d}t,\\
\mathrm{d}I_u(t) &= \left((1-\theta(t))\gamma_I I(t) - \gamma_{I_u} I_u(t)\right)\mathrm{d}t,\\
\mathrm{d}H_R(t) &= \left(\theta(t)\left(1-\frac{\omega(t)}{\theta(t)}\right)\gamma_I I(t) - \gamma_{H_R} H_R(t)\right)\mathrm{d}t,\\
\mathrm{d}H_D(t) &= \left(\omega(t)\gamma_I I(t) - \gamma_{H_D} H_D(t)\right)\mathrm{d}t,\\
\mathrm{d}R_d(t) &= \gamma_{H_R}(t)H_R(t)\mathrm{d}t,\\
\mathrm{d}R_u(t) &= \gamma_{I_u}(t)I_u(t)\mathrm{d}t,\\
\mathrm{d}D(t) &= \gamma_{H_D}(t)H_D(t)\mathrm{d}t,\\
\mathrm{d}\tilde{\beta}_I(t) &= \nu_{\beta_I}(\mu_{\beta_I} - \tilde{\beta}_I(t))\mathrm{d}t + \sigma_{\beta_I}\sqrt{\tilde{\beta}_I(t)}\mathrm{d}W(t).
\end{aligned}
$$

with $M(t)$ as defined before.

- Although only one source of randomness is introduced, the solution of the whole system becomes a set of stochastic processes.
- The remaining equations depend on the first equations for S and E, which present a dependence on $\tilde{\beta}_I$.
- The CIR process is widely employed to simulate the evolution of interest rates in quantitative finance.
- In some sense, the interest rates in finance and the disease contact rates in epidemiology present a rather similar behaviour: positiveness, controlled variability and long-term stability.

- For a set of constant initial data $S(0)$, $E(0)$, $I(0)$, $I_u(0)$, $H_R(0)$, $H_D(0)$, $R_d(0)$, $R_u(0)$, $D(0)$ and $\tilde{\beta}_I(0)$, the system of SDEs has a unique strong solution.
- As the system of SDEs is nonlinear, it is not possible to obtain a closed-form expression for the solution.
- The use of numerical methods becomes mandatory. We adopt the following strategy:
  1. Perform a simulation of the dynamics of $\tilde{\beta}_I(t)$, in accordance with the CIR process.
  2. Solve the resulting ODE system for each path of $\tilde{\beta}_I(t)$.
- We obtain a set of random walks for each stochastic process representing a model variable.

- The CIR process is a well-studied dynamics often employed in computational finance.
- The underlying distribution is known analytically, relying on the *non-central chi-squared* distribution.
- Given two time points, s and t, $s < t$, the conditional distribution of $\tilde{\beta}_I$ reads

$$\tilde{\beta}_I(t)|\tilde{\beta}_I(s) \sim c(t,s) \cdot \chi^2\left(d, \frac{\mathrm{e}^{-\nu_{\beta_I}(t-s)}}{c(t,s)}\tilde{\beta}_I(s)\right),$$

where

$$c(t,s) = \frac{\sigma^2_{\beta_I}}{4\mu_{\beta_I}}\left(1-\mathrm{e}^{-\nu_{\beta_I}(t-s)}\right), \quad d = \frac{4\nu_{\beta_I}\mu_{\beta_I}}{\sigma^2_{\beta_I}},$$

and $\chi^2(a,b)$ is the non-central chi-squared distribution with a degrees of freedom and non-centrality parameter b.

- We can then define an *exact simulation* scheme, which can be used to obtain realizations of $\tilde{\beta}_I$.
- Given a set of $m+1$ time points, $\{t_i\}_0^m$, where the solution will be computed, we have, for $i = 0, \ldots, m-1$,

$$
\begin{aligned}
c(t_{i+1}, t_i) &= \frac{\sigma_{\beta_I}^2}{4\mu_{\beta_I}} \left(1 - \mathrm{e}^{-\nu_{\beta_I}(t_{i+1}-t_i)}\right), \\
\tilde{\beta}_I(t_{i+1}) &= c(t_{i+1}, t_i)\chi^2 \left(d, \frac{\mathrm{e}^{-\nu_{\beta_I}(t_{i+1}-t_i)}}{c(t_{i+1}-t_i)} \tilde{\beta}_I(t_i)\right)
\end{aligned}
$$

given some initial value $\tilde{\beta}_I(t_0) = \tilde{\beta}_I(0)$.

- By employing this scheme, we generate n simulated discrete sample paths of $\tilde{\beta}_I$.

The stochastic θ-SEIHRD model

Potential

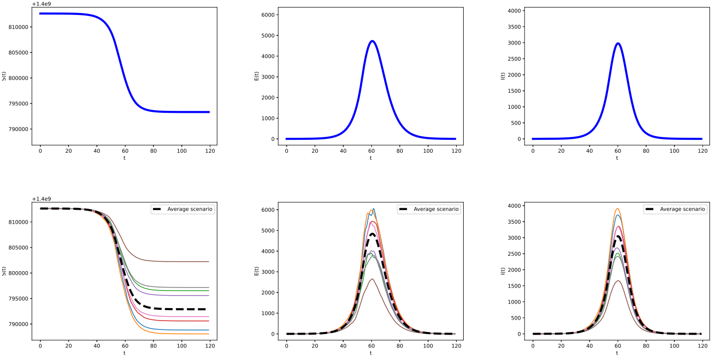


Figure: Deterministic vs. Stochastic: $\nu_{\beta_I} = 1$, $\mu_{\beta_I} = \beta_I$ and $\sigma_{\beta_I} = 0.1$, with $n = 2^{15}$ Monte Carlo simulations (only 8 simulations depicted).

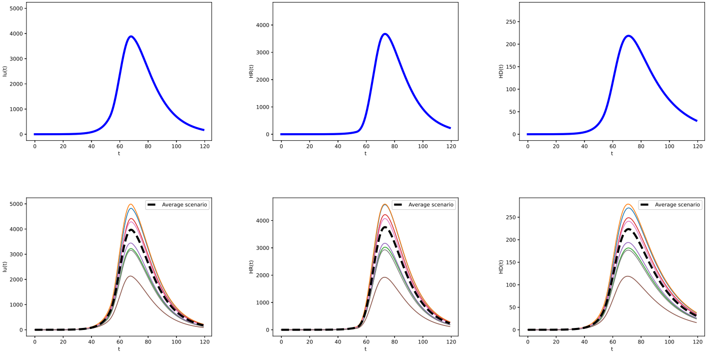


Figure: Deterministic vs. Stochastic: $\nu_{\beta_I} = 1$, $\mu_{\beta_I} = \beta_I$ and $\sigma_{\beta_I} = 0.1$, with $n = 2^{15}$ Monte Carlo simulations (only 8 simulations depicted).

The stochastic θ-SEIHRD model

Potential

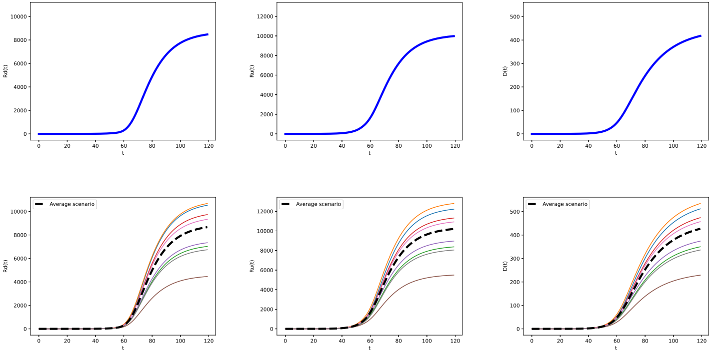


Figure: Deterministic vs. Stochastic: $\nu_{\beta_I} = 1$, $\mu_{\beta_I} = \beta_I$ and $\sigma_{\beta_I} = 0.1$, with $n = 2^{15}$ Monte Carlo simulations (only 8 simulations depicted).

- Computer system: CPU Intel Core i7-4720HQ 2.6GHz, 16GB RAM memory and GPU GeForce GTX 970M.
- The numerical codes have been implemented in Python.
- We consider a equally spaced time grid, i.e. $\Delta t := t_{i+1} - t_i, \forall i$, with time step $\Delta t = \frac{1}{6}$ (around 4 hours).
- We numerically solve n ODE systems, one for each path of $\tilde{\beta}_I$.
- We employ the explicit Runge-Kutta method of order 5(4), known as *RK45*, *RKDP*.
- Outcomes: the mean, the interquartile interval, $[Q_1, Q_3]$ and the worst case scenario (WS) at 95% confidence level.

Numerical and statistical analysis

Coefficients and parameters

- Initial state values given by: $S(t_0) = N - 1$, $E(t_0) = 1$ and

$$I(t_0) = I_u(t_0) = H_R(t_0) = H_D(t_0) = R_d(t_0) = R_u(t_0) = D(t_0) = 0.$$

Notation	Value	Description
β_I	0.2887	Disease contact rate of a person in compartment I.
C_E	0.3643	Reduction factor of the disease contact rate β_E w.r.t β_I.
C_u	0.4010	Reduction factor of the disease contact rate $\underline{\beta}_I$ w.r.t β_I.
δ_R	7.0000	Difference between days in compartment H_R and H_D.
δ_ω	0.0206	Difference between $\underline{\omega}$ and $\overline{\omega}$.
$\underline{\omega}$	0.0157	Lower bound of the fatality rate.
κ_1	0.1082	Efficiency of the control measures.

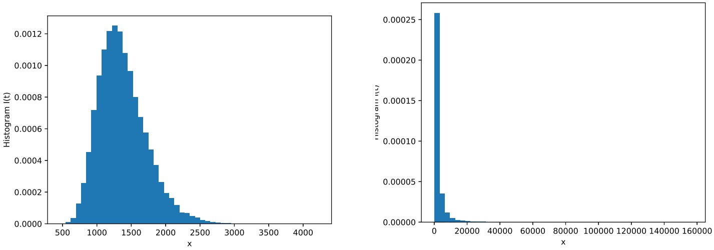
Table: Parameters obtained by calibration to the data.

Notation	Value	Description
N	1400812636	Total population.
t_0	1-12-2019	Initial date.
T	29-3-2020	Final date.
λ_1	23-1-2020	Date when travel restrictions were imposed in Wuhan.
λ_2	8-2-2020	Inflexion date.
$\underline{\theta}$	14%	Percentage of documented cases at λ_1.
$\overline{\theta}$	65%	Percentage of documented cases at λ_2.
α_H	2.75%	Percentage of infection produced by hospitalized people.
d_E	5.5	Average days in compartment E.
d_I	6.7	Average days in compartment I.
d_{I_u}	$14 - d_I = 7.3$	Average days in compartment I_u.
d_g	6	Maximum reduction of d_I due to the control measures.
C_o	14	The period of convalescence.
$p(t)$	1	Fraction of the infected people hospitalized.

Table: Parameters extracted from the experience and/or literature.

	8th February, 2020 ($t = 69$)						
	$\sigma_{\beta_I} = 0$	$\sigma_{\beta_I} = 0.1$			$\sigma_{\beta_I} = 0.5$		
	Mean	Mean	$[Q_1, Q_3]$	WS (95%)	Mean	$[Q_1, Q_3]$	WS (95%)
$E(t)$	2993	3067	[2506, 3519]	4510	5401	[1049, 5415]	18970
$I(t)$	1340	1376	[1125, 1578]	2017	2419	[476, 2423]	8522
$I_u(t)$	3854	3945	[3249, 4505]	5724	6811	[1434, 6940]	23728
$H_R(t)$	3252	3328	[2732, 3806]	4854	5799	[1182, 5863]	20340
$H_D(t)$	214	219	[181, 250]	318	377	[80, 386]	1311
$R_d(t)$	1846	1888	[1559, 2153]	2726	3231	[701, 3317]	11168
$R_u(t)$	4296	4390	[3656, 4985]	6238	7301	[1738, 7690]	24654
$D(t)$	131	134	[112, 152]	190	222	[53, 235]	747
	29th March, 2020 ($t = 119$)						
	$\sigma_{\beta_I} = 0$	$\sigma_{\beta_I} = 0.1$			$\sigma_{\beta_I} = 0.5$		
	Mean	Mean	$[Q_1, Q_3]$	WS (95%)	Mean	$[Q_1, Q_3]$	WS (95%)
$E(t)$	1	1	[1, 1]	2	2	[0, 3]	10
$I(t)$	0	0	[0, 0]	0	0	[0, 0]	1
$I_u(t)$	173	177	[146, 203]	259	309	[63, 314]	1075
$H_R(t)$	232	237	[194, 272]	348	416	[83, 420]	1458
$H_D(t)$	30	30	[25, 35]	45	53	[11, 54]	186
$R_d(t)$	8460	8662	[7118, 9910]	12624	15087	[3101, 15287]	52651
$R_u(t)$	9969	10198	[8442, 11616]	14681	17386	[3862, 17941]	59616
$D(t)$	417	426	[353, 486]	614	728	[161, 751]	2502

Table: Stochastic θ-SEIHRD model: $\nu_{\beta_I} = 1$, $\mu_{\beta_I} = \beta_I$ and $n = 2^{15}$ Monte Carlo simulations. Columns: mean, interquartile interval ($[Q_1, Q_3]$) and worst case scenario (WS).



(a) $\sigma_{\beta_I} = 0.1$ (b) $\sigma_{\beta_I} = 0.5$

Figure: Histogram of $I(t)$. Setting: $\nu_{\beta_I} = 1$ and $\mu_{\beta_I} = \beta_I$, with $n = 2^{15}$ Monte Carlo simulations.

- The cumulative number of COVID-19 cases, $c_m(t)$, at time t:

$$c_m(t) = H_R(t)+H_D(t)+R_d(t)+D(t) = c_m(t_0)+\int_{t_0}^{t} \theta(s)\gamma_I(s)I(s)\mathrm{d}s.$$

- The cumulative number of deaths, at time t: $d_m(t) = D(t)$.
- The basic reproduction number, R_0, and the effective reproduction number, $R_e(t)$, at time t, where $R_0 = R_e(t_0)$,

$$R_e(t) = \frac{U_e(t)}{\gamma_E\gamma_I(t)\gamma_{H_R}(t)\gamma_{H_D}(t)\gamma_{I_u}(t)}\frac{S(t)}{N},$$

with

$$\begin{aligned}U_e(t) = &\left(\left(\left(m_{I_u}\beta_{I_u}(1-\theta)\gamma_{H_R} + m_{H_R}\beta_{H_R}\gamma_{I_u}(\theta-\omega)\right)\right)\gamma_I + m_I\beta_I\gamma_{H_R}\gamma_{I_u}\right)\gamma_E \\ &+ m_E\beta_E\gamma_I\gamma_{H_R}\gamma_{I_u}\gamma_{H_D} + m_{H_D}\beta_{H_D}\omega\gamma_E\gamma_I\gamma_{H_R}\gamma_{I_u}.\end{aligned}$$

- Hospitalized people, $\mathrm{Hos}(t)$, at time t:

$$\mathrm{Hos}(t) = H_D(t) + p(t)\left(H_R(t) + R_d(t) - R_d(t-C_o)\right).$$

- Maximum number of hospitalized people in the interval $[t_0, t]$:

$$\mathrm{MHos}(t) = \max_{\tau \in [t_0, t]} \mathrm{Hos}(\tau).$$

- The number of individuals infected by others belonging to compartments E, I_u and $H = H_R + H_D$:

$$\Gamma_E(t) = \int_{t_0}^{t} m_E(s)\beta_E E(s)\frac{S(s)}{N}\mathrm{d}s,$$

$$\Gamma_{I_u}(t) = \int_{t_0}^{t} m_{I_u}(s)\beta_{I_u} I_u(s)\frac{S(s)}{N}\mathrm{d}s,$$

$$\Gamma_H(t) = \int_{t_0}^{t} (m_{H_R}(s)\beta_{H_R} H_R(s) + m_{H_D}(s)\beta_{H_D} H_D(s))\frac{S(s)}{N}\mathrm{d}s,$$

respectively.

Numerical and statistical analysis

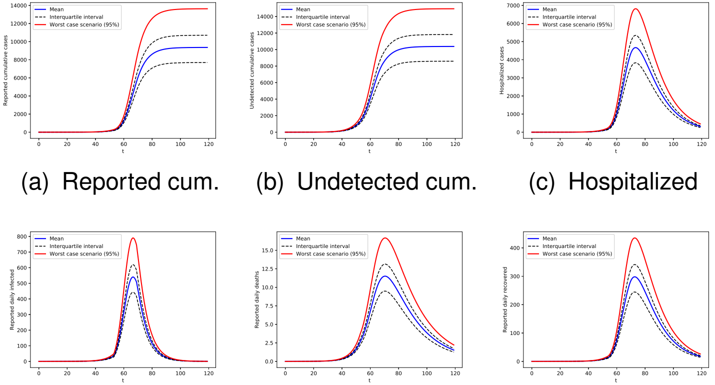
Model outputs

	8th February, 2020 ($t = 69$)						
	$\sigma_{\beta_I} = 0$	$\sigma_{\beta_I} = 0.1$			$\sigma_{\beta_I} = 0.5$		
	Mean	Mean	$[Q_1, Q_3]$	WS (95%)	Mean	$[Q_1, Q_3]$	WS (95%
$c_m(t)$	5440	5571	[4586, 6362]	8088	9631	[2026, 9812]	33466
$d_m(t)$	131	134	[112, 152]	190	222	[53, 235]	747
$R_e(t)$	0.33	0.33	[0.31, 0.35]	0.38	0.33	[0.22, 0.41]	0.63
$Hos(t)$	4040	4134	[3395, 4727]	6026	7197	[1471, 7273]	25262
MHos(t)	4040	4134	[3395, 4727]	6026	7197	[1471, 7273]	25262
$\Gamma_E(t)$	5012	5126	[4255, 5833]	7337	8625	[1979, 9013]	29414
$\Gamma_{I_u}(t)$	4550	4646	[3864, 5285]	6640	7600	[1764, 7976]	25755
$\Gamma_H(t)$	198	202	[168, 230]	288	328	[78, 346]	1106
	29th March, 2020 ($t = 119$)						
	$\sigma_{\beta_I} = 0$	$\sigma_{\beta_I} = 0.1$			$\sigma_{\beta_I} = 0.5$		
	Mean	Mean	$[Q_1, Q_3]$	WS (95%)	Mean	$[Q_1, Q_3]$	WS (95%
$c_m(t)$	9140	9358	[7691, 10704]	13631	16286	[3358, 16526]	56752
$d_m(t)$	417	426	[353, 486]	614	728	[161, 751]	2502
$R_e(t)$	0.0	0.0	[0.0 , 0.0]	0.0	0.0	[0.0, 0.0]	0.0
Hos(t)	306	314	[257, 360]	459	549	[111, 555]	1927
MHos(t)	4558	4671	[3832, 5347]	6816	8195	[1662, 8258]	28681
$\Gamma_E(t)$	5259	5379	[4464, 6122]	7705	9070	[2073, 9465]	30925
$\Gamma_{I_u}(t)$	5388	5504	[4570, 6264]	7886	9082	[2080, 9479]	30911
$\Gamma_H(t)$	229	234	[195, 266]	334	384	[89, 402]	1298

Table: Stochastic θ-SEIHRD model: $\nu_{\beta_I} = 1$, $\mu_{\beta_I} = \beta_I$ and $n = 2^{15}$ Monte Carlo simulations. Columns: mean, interquartile interval ($[Q_1, Q_3]$) and worst case scenario (WS).

Numerical and statistical analysis

Evolution epidemic curves



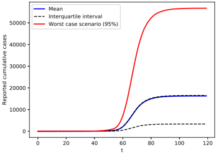
(a) Reported cum. (b) Undetected cum. (c) Hospitalized

(d) Daily infected (e) Daily deaths (f) Daily recovered

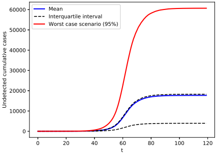
Figure: Epidemic curves: mean, IQ interval and WS. Setting: $\nu_{\beta_I} = 1$, $\mu_{\beta_I} = \beta_I$ and $\sigma_{\beta_I} = 0.1$, with $n = 2^{15}$.

Numerical and statistical analysis

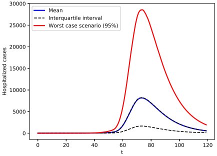
Evolution epidemic curves



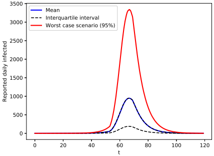
(a) Reported cum.



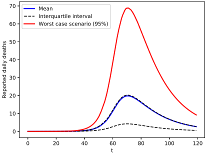
(b) Undetected cum.



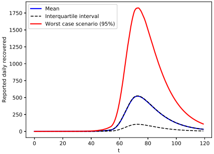
(c) Hospitalized



(d) Daily infected



(e) Daily deaths



(f) Daily recovered

Figure: Epidemic curves: mean, IQ interval and WS. Setting: $\nu_{\beta_I} = 1$, $\mu_{\beta_I} = \beta_I$ and $\sigma_{\beta_I} = 0.5$, with $n = 2^{15}$.

Conclusions

Discussion and and future research

- We have extended an *ad-hoc* model by incorporating randomness to relevant coefficients (disease contact rates).
- We have shown the importance of considering the uncertainty.
- The presented modelling approach is more complete since it allows to compute confidence intervals and worst case scenarios.
- The information provided by the worst case scenarios can be useful to develop more conservative policies in the actions against the COVID-19 spread.
- A natural extension would be to consider independent contact rates, using different stochastic processes to characterize their dynamics.
- In this more general setting, a certain number of different (possibly correlated) Brownian motion processes would come into place.

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Thank you for your attention

More: `alvaroleitao.github.io`