Efficient one and multiple time-step simulation of the SABR model

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"Our" definition of simulation

- Generate samples from (sampling) stochastic processes.
- The standard approach to sample from a given distribution, Z:

$$F_Z(Z) \stackrel{\mathrm{d}}{=} U \quad \text{thus} \quad z_n = F_Z^{-1}(u_n),$$

- F_Z is the cumulative distribution function (CDF).
- $\stackrel{\mathrm{d}}{=}$ means equality in the distribution sense.
- $U \sim \mathcal{U}([0,1])$ and u_n is a sample from $\mathcal{U}([0,1])$.
- The computational cost depends on inversion F_Z^{-1}.

Outline

SABR model

- The formal definition of the SABR model [5] reads

$$
\begin{aligned}
\mathrm{d}S(t) &= \sigma(t)S^{\beta}(t)\mathrm{d}W_S(t), \quad & S(0) &= S_0 \exp\left(rT\right), \\
\mathrm{d}\sigma(t) &= \alpha\sigma(t)\mathrm{d}W_\sigma(t), \quad & \sigma(0) &= \sigma_0.
\end{aligned}
$$

- $S(t) = \bar{S}(t)\exp\left(r(T-t)\right)$ is the forward price of the underlying $\bar{S}(t)$, with r an interest rate, S_0 the spot price and T the maturity.
- $\sigma(t)$ is the stochastic volatility.
- $W_f(t)$ and $W_\sigma(t)$ are two correlated Brownian motions.
- SABR parameters:
  - The volatility of the volatility, $\alpha > 0$.
  - The CEV elasticity, $0 \leq \beta \leq 1$.
  - The correlation coefficient, ρ ($W_f W_\sigma = \rho t$).

"Exact" simulation of SABR model

- Based on Islah [7], the conditional cumulative distribution function (CDF) of $S(t)$ in a generic interval $[s, t]$, $0 \leq s \leq t \leq T$:

$$Pr\left(S(t) \leq K | S(s) > 0, \sigma(s), \sigma(t), \int_s^t \sigma^2(z)\mathrm{d}z\right) = 1 - \chi^2(a; b, c),$$

where

$$a = \frac{1}{\nu(t)}\left(\frac{S(s)^{1-\beta}}{(1-\beta)} + \frac{\rho}{\alpha}\left(\sigma(t) - \sigma(s)\right)\right)^2,$$

$$c = \frac{K^{2(1-\beta)}}{(1-\beta)^2\nu(t)},$$

$$b = 2 - \frac{1 - 2\beta - \rho^2(1-\beta)}{(1-\beta)(1-\rho^2)},$$

$$\nu(t) = (1-\rho^2)\int_s^t \sigma^2(z)\mathrm{d}z,$$

and $\chi^2(x; \delta, \lambda)$ is the non-central chi-square CDF.

- Exact in the case of $\rho = 0$, an *approximation* otherwise.

Simulation of SABR model

- Simulation of the volatility process, $\sigma(t)|\sigma(s)$:

$$\sigma(t) \sim \sigma(s)\exp(\alpha \hat{W}_\sigma(t) - \frac{1}{2}\alpha^2 t),$$

  where $\hat{W}_\sigma(t)$ is a independent Brownian motion.
- Simulation of the integrated variance process, $\int_s^t \sigma^2(z)\mathrm{d}z|\sigma(t), \sigma(s)$.
- Simulation of the forward process, $S(t)|S(s), \int_s^t \sigma^2(z)\mathrm{d}z, \sigma(t), \sigma(s)$ by inverting the CDF.
- The conditional integrated variance is a challenging part. We propose:
  - Approximate the conditional distribution by using Fourier techniques and copulas.
  - Marginal distribution based on COS method [3].
  - Conditional distribution based on copulas.
  - Improvements in performance and efficiency.

Distribution of the integrated variance

- Not available.
- For notational convenience, we will use $Y(s,t) := \int_s^t \sigma^2(z)\mathrm{d}z$.
- Discrete equivalent, M monitoring dates:

$$Y(s,t) := \int_s^t \sigma^2(z)\mathrm{d}z \approx \sum_{j=1}^{M} \Delta t \sigma^2(t_j) =: \hat{Y}(s,t)$$

where $t_j = s + j\Delta t$, $j = 1, \ldots, M$ and $\Delta t = \frac{t-s}{M}$.

- In the logarithmic domain, where we aim to find an approximation of $F_{\log \hat{Y}|\log\sigma(s)}$:

$$F_{\log \hat{Y}|\log\sigma(s)}(x) = \int_{-\infty}^{x} f_{\log \hat{Y}|\log\sigma(s)}(y)\mathrm{d}y,$$

where $f_{\log \hat{Y}|\log\sigma(s)}$ is the *probability density function* (PDF) of $\log \hat{Y}(s,t)|\log\sigma(s)$.

PDF of the integrated variance

- Equivalent: Characteristic function and inversion (Fourier pair).
- Recursive procedure to derive an approximated $\phi_{\log \hat{Y}|\log \sigma(s)}$.
- We start by defining the logarithmic increment of $\sigma^2(t)$:

$$R_j = \log\left(\frac{\sigma^2(t_j)}{\sigma^2(t_{j-1})}\right), j = 1, \ldots, M.$$

- $\sigma^2(t_j)$ can be written:

$$\sigma^2(t_j) = \sigma^2(t_0)\exp(R_1 + R_2 + \cdots + R_j).$$

- We introduce the iterative process

$$\begin{aligned} Y_1 &= R_M, \\ Y_j &= R_{M+1-j} + Z_{j-1}, \quad j = 2, \ldots, M. \end{aligned}$$

with $Z_j = \log(1 + \exp(Y_j))$.

PDF of the integrated variance (cont.)

- $\hat{Y}(s,t)$ can be expressed:

$$\hat{Y}(s,t) = \sum_{i=1}^{M} \sigma^2(t_i)\Delta t = \Delta t \sigma^2(s) \exp(Y_M).$$

- And, we compute $\phi_{\log \hat{Y} | \log \sigma(s)}(u)$, as follows:

$$\phi_{\log \hat{Y} | \log \sigma(s)}(u) = \exp\left(iu \log(\Delta t \sigma^2(s))\right) \phi_{Y_M}(u).$$

- By applying COS method [3] in the support $[\hat{a}, \hat{b}]$:

$$f_{\log \hat{Y} | \log \sigma(s)}(x) \approx \frac{2}{\hat{b}-\hat{a}} \sum_{k=0}^{N-1}{}' C_k \cos\left((x-\hat{a})\frac{k\pi}{\hat{b}-\hat{a}}\right),$$

with

$$C_k = \Re\left(\phi_{\log \hat{Y} | \log \sigma(s)}\left(\frac{k\pi}{\hat{b}-\hat{a}}\right) \exp\left(-i\frac{\hat{a}k\pi}{\hat{b}-\hat{a}}\right)\right).$$

CDF of the integrated variance

- The CDF of $\log \hat{Y}(s,t) | \log \sigma(s)$:

$$F_{\log \hat{Y} | \log \sigma(s)}(x) = \int_{-\infty}^{x} f_{\log \hat{Y} | \log \sigma(s)}(y) \mathrm{d}y$$
$$\approx \int_{\hat{a}}^{x} \frac{2}{\hat{b} - \hat{a}} \sum_{k=0}^{N-1'} C_k \cos\left((y - \hat{a}) \frac{k\pi}{\hat{b} - \hat{a}} \right) \mathrm{d}y.$$

- The efficient computation of $\phi_{\log \hat{Y} | \log \sigma(s)}$ is crucial for the performance of the whole procedure (specially, one-step case).
- The inversion of $F_{\log \hat{Y} | \log \sigma(s)}$ is relatively expensive (unafforable in the multi-step case).

Copula-based simulation of $\int_s^t \sigma^2(z)\mathrm{d}z|\sigma(t), \sigma(s)$

- In order to apply copulas, we need (logarithmic domain):
  - $F_{\log \hat{Y}|\log \sigma(s)}$.
  - $F_{\log \sigma(t)|\log \sigma(s)}$.
  - Correlation between $\log Y(s,t)$ and $\log \sigma(t)$.
- The distribution of $\log \sigma(t)|\log \sigma(s)$ is known ($\sigma(t)$ follows a log-normal distribution).
- Approximated Pearson's correlation coefficient:

$$\mathcal{P}_{\log Y, \log \sigma(t)} \approx \frac{t^2 - s^2}{2\sqrt{\left(\frac{1}{3}t^4 + \frac{2}{3}ts^3 - t^2s^2\right)}}.$$

- For some copulas, like Archimedean, Kendall's τ is required:

$$\mathcal{P} = \sin\left(\frac{\pi}{2}\tau\right).$$

Sampling $\int_s^t \sigma^2(z)\mathrm{d}z|\sigma(t), \sigma(s)$: Steps

1. Determine $F_{\log\sigma(t)|\log\sigma(s)}$ and $F_{\log\hat{Y}|\log\sigma(s)}$.
2. Determine the correlation between $\log Y(s,t)$ and $\log\sigma(t)$.
3. Generate correlated uniform samples, $U_{\log\sigma(t)|\log\sigma(s)}$ and $U_{\log\hat{Y}|\log\sigma(s)}$ by means of copula.
4. From $U_{\log\sigma(t)|\log\sigma(s)}$ and $U_{\log\hat{Y}|\log\sigma(s)}$ invert original marginal distributions.
5. The samples of $\sigma(t)|\sigma(s)$ and $Y(s,t) = \int_s^t \sigma^2(z)\mathrm{d}z|\sigma(t), \sigma(s)$ are obtained by taking exponentials.

One time-step simulation of the SABR model

- $s = 0$ and $t = T$, with T the maturity time.
- The use is restricted to price European options up to $T = 2$.
- $\log \sigma(s)$ becomes constant.
- $F_{\log \sigma(t) | \log \sigma(s)}$ and $F_{\log \hat{Y} | \log \sigma(s)}$ turn into $F_{\log \sigma(T)}$ and $F_{\log \hat{Y}(T)}$.
- The computation of $\phi_{\log \hat{Y}(T)}$ is much simpler and very fast.
- The approximated Pearson's coefficient results in a constant value:

$$\mathcal{P}_{\log Y(T), \log \sigma(T)} \approx \frac{T^2}{2\sqrt{\frac{1}{3}T^4}} = \frac{\sqrt{3}}{2}.$$

Approximated correlation

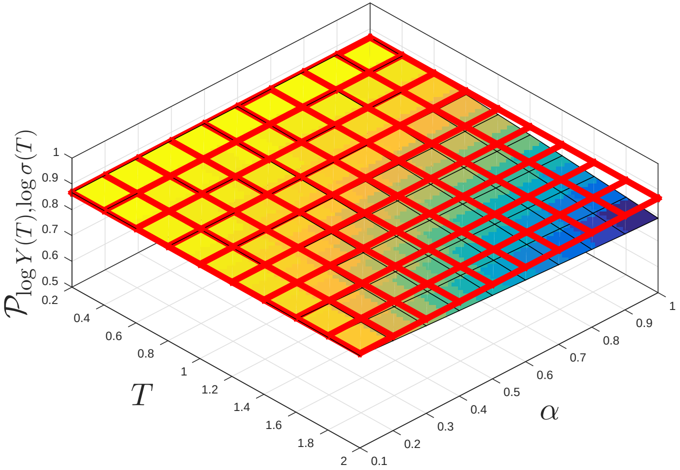


Figure: Pearson's coefficient: Empirical (surface) vs. approximation (red grid).

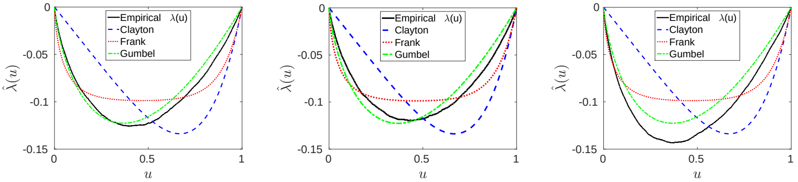
Copula analysis

- Based on the one-step simulation, a copula analysis is carried out.
- Gaussian, Student **t** and Archimedean (Clayton, Frank and Gumbel).
- A *goodness-of-fit (GOF)* for copulas needs to be evaluated.
- Archimedean: graphic GOF based on Kendall's processes.
- Generic GOF based on the so-called *Deheuvels or empirical copula*.

	S_0	σ_0	α	β	ρ	T
Set I	1.0	0.5	0.4	0.7	0.0	2
Set II	0.05	0.1	0.4	0.0	−0.8	0.5
Set III	0.04	0.4	0.8	1.0	−0.5	2

Table: Data sets.

GOF - Archimedean



(a) Set I. (b) Set II. (c) Set III.

Figure: Archimedean GOF test: $\hat{\lambda}(u)$ vs. empirical $\lambda(u)$.

	Clayton	Frank	Gumbel
Set I	1.3469×10^{-3}	2.9909×10^{-4}	5.1723×10^{-5}
Set II	1.0885×10^{-3}	2.1249×10^{-4}	8.4834×10^{-5}
Set III	2.1151×10^{-3}	7.5271×10^{-4}	2.6664×10^{-4}

Table: MSE of $\hat{\lambda}(u) - \lambda(u)$.

Generic GOF

	Gaussian	Student $\mathbf{t}$ ($\nu = 5$)	Gumbel
Set I	5.0323×10^{-3}	5.0242×10^{-3}	3.8063×10^{-3}
Set II	3.1049×10^{-3}	3.0659×10^{-3}	4.5703×10^{-3}
Set III	5.9439×10^{-3}	6.0041×10^{-3}	4.3210×10^{-3}

Table: Generic GOF: D_2.

- The three copulas perform very similarly.
- For longer maturities: Gumbel performs better.
- The Student **t** copula is discarded: very similar to the Gaussian copula and the calibration of the ν parameter adds extra complexity.
- As a general strategy, the Gumbel copula is the most robust choice.
- With short maturities, the Gaussian copula may be a satisfactory alternative.

Pricing European options

- The strike values K_i are chosen following the expression:

$$K_i(T) = S(0)\exp(0.1 \times T \times \delta_i),$$
$$\delta_i = -1.5, -1.0, -0.5, 0.0, 0.5, 1.0, 1.5.$$

- Forward asset, $S(t)$: enhanced inversion by Chen et al. [2].
- Martingale correction:

$$S(t) = S(t) - \frac{1}{n}\sum_{i=1}^{n} S_i(t) + \mathbb{E}[S(t)],$$
$$= S(t) - \frac{1}{n}\sum_{i=1}^{n} S_i(t) + S_0,$$

Pricing European options - Convergence and time

	$n = 1000$	$n = 10000$	$n = 100000$	$n = 1000000$
	Gaussian (Set I, K_1)			
Error	519.58	132.39	37.42	16.23
Time	0.3386	0.3440	0.3857	0.5733
	Gumbel (Set I, K_1)			
Error	151.44	-123.76	34.14	11.59
Time	0.3492	0.3561	0.3874	0.6663

Table: Convergence in number of samples, n: error (basis points) and execution time (*sec.*).

Pricing European options - Implied volatilities

Strikes	K_1	K_2	K_3	K_4	K_5	K_6	K_7
	Set I (Reference: Antonov [1])						
Hagan	55.07	52.34	50.08	N/A	47.04	46.26	45.97
MC	23.50	21.41	19.38	N/A	16.59	15.58	14.63
Gaussian	16.23	20.79	24.95	N/A	33.40	37.03	40.72
Gumbel	11.59	15.57	19.12	N/A	25.41	28.66	31.79
	Set II (Reference: Korn [8])						
Hagan	−558.82	−492.37	−432.11	−377.47	−327.92	−282.98	−242.22
MC	5.30	6.50	7.85	9.32	10.82	12.25	13.66
Gaussian	9.93	9.98	10.02	10.20	10.57	10.73	11.04
Gumbel	−9.93	−9.38	−8.94	−8.35	−7.69	−6.83	−5.79
	Set III (Reference: MC Milstein)						
Hagan	287.05	252.91	220.39	190.36	163.87	141.88	126.39
Gaussian	16.10	16.76	16.62	15.22	13.85	12.29	10.67
Gumbel	6.99	3.79	0.67	−2.27	−5.57	−9.79	−14.06

Table: Implied volatility: errors in basis points.

- One-step SABR simulation is a fast alternative to Hagan formula.
- Overcomes the known issues, like low strikes and high volatilities.
- For longer maturities and more complex options: multiple time-step

Multiple time-step simulation of the SABR model

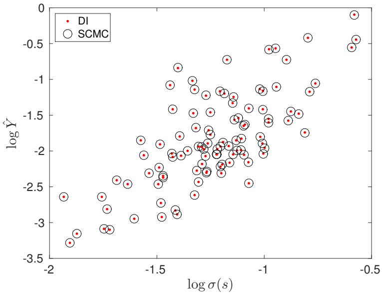
- We denote it *mSABR* simulation method (scheme).
- In intermediate steps, $\phi_{\log \hat{Y}|\log \sigma(s)}$ becomes "stochastic".
- $f_{\log \hat{Y}|\log \sigma(s)}$ needs to be computed for each sample of $\log \sigma(s)$.
- Consequently, the inversion of $F_{\log \hat{Y}|\log \sigma(s)}$ is unafforable ($n \uparrow\uparrow$).
- Solution: *Stochastic Collocation Monte Carlo* (SCMC) sampler [4].

$$y_n|v_n \approx g_{L_{\hat{Y}},L_\sigma}(x_n) = \sum_{i=1}^{L_{\hat{Y}}} \sum_{j=1}^{L_\sigma} F^{-1}_{\log \hat{Y}|\log \sigma(s)=v_j}(F_X(x_i))\ell_i(x_n)\ell_j(v_n),$$

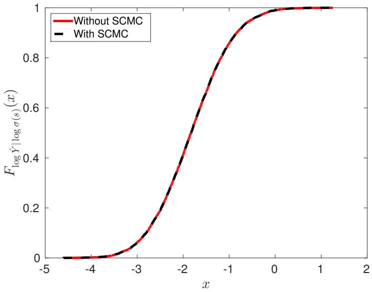
where x_n are the samples from the *cheap variable*, X, and v_n the given samples of $\log \sigma(s)$. x_i and v_j are the *collocation points* of X and $\log \sigma(s)$, respectively. ℓ_i and ℓ_j are the Lagrange polynomials defined by

$$\ell_i(x_n) = \prod_{k=1,k\neq i}^{L_{\hat{Y}}} \frac{x_n - x_k}{x_i - x_k}, \quad \ell_j(v_n) = \prod_{k=1,k\neq j}^{L_\sigma} \frac{v_n - v_k}{v_i - v_k}.$$

Application of 2D SCMC to $F_{\log \hat{Y} | \log \sigma(s)}$



(a) $\log \hat{Y} | \log \sigma(s)$ - DI. vs. SCMC.



(b) $F_{\log \hat{Y} | \log \sigma(s)}(x)$.

Samples	Without SCMC	With SCMC		
		$L_{\hat{Y}} = L_\sigma = 3$	$L_{\hat{Y}} = L_\sigma = 7$	$L_{\hat{Y}} = L_\sigma = 11$
100	1.0695	0.0449	0.0466	0.0660
10000	16.3483	0.0518	0.0588	0.0798
1000000	1624.3019	0.2648	0.5882	1.0940

Table: SCMC time

mSABR method - Experiments

- The strike values K_i are chosen following the expression:

$$K_i(T) = S(0)\exp(0.1 \times T \times \delta_i),$$
$$\delta_i = -1.5, -1.0, -0.5, 0.0, 0.5, 1.0, 1.5.$$

- Forward asset, $S(t)$: enhanced inversion by Chen et al. [2].
- Martingale correction:

$$S(t) = S(t) - \frac{1}{n}\sum_{i=1}^{n} S_i(t) + S_0,$$

- New data sets:

	S_0	σ_0	α	β	ρ	T
Set I [4]	0.5	0.5	0.4	0.5	0.0	4
Set II [2]	0.04	0.2	0.3	1.0	-0.5	5
Set III [1]	1.0	0.25	0.3	0.6	-0.5	20
Set IV [6]	2.0	0.35	1.0	0.0	0.0	1

Table: Data sets.

mSABR method - Convergence test I

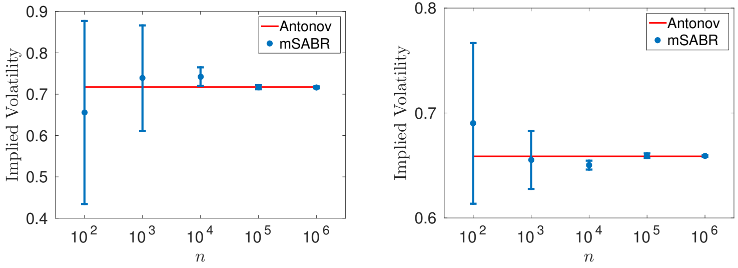
- Convergence in number of time-steps, m: Antonov vs. mSABR. Set I.

Strikes	K_1	K_2	K_3	K_4	K_5	K_6	K_7
Antonov	73.34%	71.73%	70.17%	N/A	67.23%	65.87%	64.59%
$m = T/4$	71.34%	69.98%	68.65%	67.36%	66.11%	64.92%	63.80%
Error(bp)	−199.96	−174.93	−151.81	N/A	−111.81	−94.64	−78.93
$m = T/2$	71.90%	70.41%	68.95%	67.54%	66.20%	64.91%	63.70%
Error(bp)	−143.63	−132.23	−121.75	N/A	−103.55	−95.89	−89.16
$m = T$	73.05%	71.46%	69.92%	68.45%	67.03%	65.68%	64.42%
Error(bp)	−28.72	−26.40	−24.28	N/A	−20.28	−18.61	−17.10
$m = 2T$	73.24%	71.62%	70.06%	68.55%	67.11%	65.74%	64.45%
Error(bp)	−10.21	−10.66	−11.05	N/A	−12.29	−13.03	−14.07
$m = 12T$	73.38%	71.76%	70.19%	68.69%	67.25%	65.88%	64.59%
Error(bp)	4.25	3.52	2.73	N/A	1.55	0.88	0.22

mSABR method - Convergence test II

- Convergence in number of samples, n: Antonov vs. mSABR. Set I.

Strikes	K_1	K_2	K_3	K_4	K_5	K_6	K_7
Antonov	73.34%	71.73%	70.17%	N/A	67.23%	65.87%	64.59%
$n = 10^2$	82.26%	80.72%	79.20%	77.68%	76.12%	74.56%	73.52%
RE	1.21×10^{-1}	1.25×10^{-1}	1.28×10^{-1}	N/A	1.32×10^{-1}	1.31×10^{-1}	1.38×10^{-1}
$n = 10^4$	74.05%	72.52%	71.01%	69.54%	68.12%	66.73%	65.40%
RE	9.71×10^{-3}	1.10×10^{-2}	1.20×10^{-2}	N/A	1.31×10^{-2}	1.30×10^{-2}	1.25×10^{-2}
$n = 10^6$	73.38%	71.76%	70.19%	68.69%	67.25%	65.88%	64.59%
RE	5.79×10^{-4}	4.90×10^{-4}	3.89×10^{-4}	N/A	2.30×10^{-4}	1.33×10^{-4}	3.40×10^{-5}



(c) Strike K_2. (d) Strike K_6.

mSABR method - Pricing barrier options

- The *up-and-out* call option is considered here
- The price, with the barrier level, B, $B > S_0, B > K_i$, reads:

$$V_i(K_i, B, T) = \exp(-rT)\,\mathbb{E}\left[(S(T) - K_i)\mathbb{1}\left(\max_{0<t_k\leq T} S(t_k) > B\right)\right],$$

where t_k are the times where the barrier condition is checked.

- We set $n = 10^6$ and $m = 12T$.
- We define the *mean squared error* (MSE) as

$$\mathrm{MSE} = \frac{1}{7}\sum_{i=1}^{7}\left(V_i^{MC}(K_i, B, T) - V_i^{mSABR}(K_i, B, T)\right)^2$$

where $V_i^{MC}(K_i, B, T)$ and $V_i^{mSABR}(K_i, B, T)$ are the barrier option prices provided by standard Monte Carlo method and by the mSABR method, respectively.

mSABR method - Pricing barrier options

- Pricing barrier options with mSABR: $V_i(K_i, B, T) \times 100$. Set I:

Strikes	K_1	K_2	K_3	K_4	K_5	K_6	K_7
	$B = 0.08$)						
MC	1.1588	0.9361	0.7172	0.5127	0.3345	0.1931	0.0938
mSABR	1.1592	0.9363	0.7173	0.5128	0.3345	0.1931	0.0938
MSE	2.9029×10^{-12}						
	$B = 0.1$						
MC	1.3056	1.0726	0.8421	0.6249	0.4326	0.2757	0.1595
mSABR	1.3011	1.0685	0.8386	0.6218	0.4299	0.2735	0.1579
MSE	1.0515×10^{-9}						
	$B = 0.12$						
MC	1.3483	1.1131	0.8805	0.6605	0.4651	0.3049	0.1854
mSABR	1.3494	1.1139	0.8809	0.6609	0.4657	0.3054	0.1855
MSE	4.1112×10^{-11}						

- Pricing barrier options with mSABR: $V_i(K_i, B, T) \times 100$. Set II:

Strikes	K_1	K_2	K_3	K_4	K_5	K_6	K_7
	$B = 1.2$						
MC	5.7676	4.9768	4.1941	3.4348	2.7137	2.0478	1.4553
mSABR	5.7569	4.9692	4.1885	3.4309	2.7125	2.0485	1.4571
MSE	3.2203×10^{-9}						
	$B = 1.5$						
MC	9.6048	8.5659	7.5154	6.4675	5.4385	4.4473	3.5150
mSABR	9.5785	8.5422	7.4951	6.4506	5.4243	4.4360	3.5053
MSE	3.3896×10^{-8}						
	$B = 1.8$						
MC	13.0040	11.7995	10.5692	9.3267	8.0888	6.8718	5.6974
mSABR	12.9586	11.7641	10.5436	9.3105	8.0799	6.8702	5.7018
MSE	6.1830×10^{-8}						

mSABR method - Pricing barrier options

- Pricing barrier options with mSABR: $V_i(K_i, B, T) \times 100$. Set III:

Strikes	K_1	K_2	K_3	K_4	K_5	K_6	K_7
				$B = 2.0$			
MC	38.5970	32.1741	24.7566	16.5997	8.5043	2.2349	0.0068
mSABR	38.8818	32.3782	24.8867	16.6718	8.5316	2.2384	0.0069
MSE	2.0806×10^{-6}						
				$B = 2.5$			
MC	54.1146	46.2997	37.1557	26.8795	16.2045	6.8740	1.3530
mSABR	54.4078	46.5148	37.3014	26.9673	16.2519	6.9029	1.3643
MSE	2.3480×10^{-6}						
				$B = 3.0$			
MC	59.4318	51.2607	41.6728	30.8467	19.4854	9.3085	2.7911
mSABR	60.0604	51.7780	42.0812	31.1475	19.6829	9.4296	2.8435
MSE	1.3948×10^{-5}						

- Pricing barrier options with mSABR: $V_i(K_i, B, T) \times 100$. Set IV:

Strikes	K_1	K_2	K_3	K_4	K_5	K_6	K_7
				$B = 2.5$			
MC	22.0200	16.2459	10.9944	6.6236	3.4153	1.4017	0.3791
mSABR	21.9453	16.1852	10.9492	6.5935	3.3991	1.3962	0.3759
MSE	1.7886×10^{-7}						
				$B = 3.0$			
MC	29.6708	23.0464	16.9160	11.6380	7.4998	4.5484	2.5930
mSABR	29.6115	23.0018	16.8864	11.6200	7.4885	4.5400	2.5873
MSE	9.8997×10^{-8}						
				$B = 3.5$			
MC	32.0136	25.2357	18.9431	13.4904	9.1668	6.0184	3.8600
mSABR	31.9417	25.1684	18.8809	13.4379	9.1226	5.9829	3.8313
MSE	2.0102×10^{-7}						

Conclusions

- We propose an efficient SABR simulation based on Fourier and copula techniques.
- The one-step SABR is a fast alternative to Hagan formula for short maturities.
- Overcomes the known issues of Hagan's expression.
- When longer maturities and/or more involved options are considered, multi-step version.
- High accuracy with very few number of time-steps.
- More details:

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Acknowledgments

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ITN STRIKE

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Thank you for your attention