Model-free computation of risk contributions in credit portfolios

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Motivation

- In a financial institution, portfolio credit risk represents one of the most important sources of risk.
- The well-known VaR and ES risk measures are usually employed.
- Besides, the decomposition of the total risk into the individual risk contribution of each obligor is a problem of practical importance.
- Identification of risk concentrations, portfolio optimization or capital allocation are, among others, relevant examples of application.
- The problem of obtaining the risk contributions represents a great challenge from the computational standpoint.
- Commonly in practice: Monte Carlo methods. Easy to implement and understand, and attractive for practitioners, but rather expensive.

What we propose

- An alternative approach for computing risk contributions based on non-parametric density estimation based on wavelets.
- Once the density function of the loss variable is recovered, we derive closed-form solutions for VaR and ES.
- According to the Euler's capital allocation principle, the risk contributions can be calculated by taking partial derivatives of the risk measures (VaR or ES) w.r.t. the individual exposures.
- Thanks to the wavelet properties, these partial derivatives can be efficiently computed, obtaining high precision.
- The presented methodology is model-free, in the sense that it applies in the same manner regardless of the model driving the losses.

Outline

1. Problem formulation
2. Risk measures and risk contributions
3. Wavelet-based estimation of the loss distribution
4. Numerical experiments
5. Conclusions

Problem formulation

- Let us consider a portfolio consisting in N obligors.
- Each obligor j is characterized by the **exposure at default**, E_j, the **probability of default**, P_j, and the **loss given default**, 100%.
- We follow the framework of Merton's firm-value model.
- Let $V_j(t)$ denote the firm value of obligor j at time $t < T$, where T is the time horizon (typically one year).
- The obligor j defaults when its value at the end of the observation period, $V_j(T)$, falls below a certain threshold, τ_j, i.e, $V_j(T) < \tau_j$.
- We can therefore define the default indicator as $D_j = \mathbb{1}_{\{V_j(T) < \tau_j\}}$.
- Given D_j, the individual loss of obligor j is defined as,

$$L_j = D_j \cdot E_j,$$

while the total loss in the portfolio reads,

$$L = \sum_{j=1}^{N} L_j.$$

Factor models

- The firm (obligor) value V_j is split into two terms: one common component called **systematic** factor, and an **idiosyncratic** component for each obligor.
- Depending on the number of factors of the systematic part, the model can be classified into the *one-* or *multi-factor* class.
- **One-factor models**: Gaussian copula and t-copula

$$V_j = \sqrt{\rho_j} Y + \sqrt{1-\rho_j}\varepsilon_j, \quad V_j = \sqrt{\frac{\nu}{W}} \left(\sqrt{\rho_j} Y + \sqrt{1-\rho_j}\varepsilon_j \right),$$

where $\varepsilon_1, \cdots, \varepsilon_N, Y \sim \mathcal{N}(0,1)$, W follows a chi-square distribution $\chi^2(\nu)$ with ν degrees of freedom and $\varepsilon_1, \cdots, \varepsilon_N$, Y and W are mutually independent. The parameters $\rho_1, \cdots, \rho_j \in (0,1)$ are the correlation coefficients.

Factor models

- When we need to capture complicated correlation structures, extend the previous models to multiple dimensions.
- **Multi-factor models**:

$$V_j = \boldsymbol{a_j}^T \boldsymbol{Y} + b_j \varepsilon_j, \quad j = 1, \cdots, N.$$

where $\boldsymbol{Y} = [Y_1, \; Y_2, \dots, Y_d]^T$ denotes the systematic risk factors. Here, $\boldsymbol{a_j} = [a_{j1}, a_{j2}, \dots, a_{jd}]^T$ represents the factor loadings satisfying $\boldsymbol{a_j}^T \boldsymbol{a_j} < 1$, and b_j, being the factor loading of the idiosyncratic risk factor, $b_j = \sqrt{1 - \left(a_{j1}^2 + a_{j2}^2 + \cdots + a_{jd}^2\right)}$, ensuring $V_j \sim \mathcal{N}(0, 1)$.

- Similarly, the multi-factor *t*-copula model definition reads,

$$V_j = \sqrt{\frac{\nu}{W}} \left(\boldsymbol{a_j}^T \boldsymbol{Y} + b_j \varepsilon_j\right), \quad j = 1, \cdots, N,$$

where $\boldsymbol{Y}, \varepsilon_j, \boldsymbol{a_j}$ and b_j are defined as before, with $W \sim \chi^2(\nu)$.

Risk measures

- We will use two well-known measures of risk, the **value-at-risk**, VaR, and the **expected shortfall**, ES.

Definition

Given a confidence level $\alpha \in (0, 1)$ and the vector of exposures $\boldsymbol{E} = [E_1, E_2, \ldots, E_N]^T$, we define the portfolio VaR,

$$\mathrm{VaR}_\alpha(\boldsymbol{E}) = \inf\{l \in \mathbb{R} : \mathbb{P}(L \leq l) \geq \alpha\} = \inf\{l \in \mathbb{R} : F_L(l; \boldsymbol{E}) \geq \alpha\},$$

where F_L is the distribution function of the total loss random variable L (we emphasize the dependence of VaR with respect to the risk exposures).

Risk measures

Definition

Given the loss variable L with $\mathbb{E}[|L|] < \infty$ and distribution function F_L, the ES at confidence level $\alpha \in (0,1)$ is defined as,

$$\mathrm{ES}_\alpha(\boldsymbol{E}) = \frac{1}{1-\alpha}\int_\alpha^1 \mathrm{VaR}_u(\boldsymbol{E})\mathrm{d}u.$$

- When the loss variable is integrable with continuous distribution function, then the ES satisfies the equation,

$$\mathrm{ES}_\alpha(\boldsymbol{E}) = \mathbb{E}[L | L \geq \mathrm{VaR}_\alpha(\boldsymbol{E})],$$

or, in integral form,

$$\mathrm{ES}_\alpha(\boldsymbol{E}) = \frac{1}{1-\alpha}\int_{\mathrm{VaR}_\alpha(\boldsymbol{E})}^{+\infty} x f_L(x; \boldsymbol{E})\mathrm{d}x,$$

where f_L is the probability density function of the total loss random variable L.

Risk contributions

- The goal is allocating the risk to the elements of the portfolio, based on their individual contribution to the risk measure.
- This problem is also known as *capital allocation* and a solution to this problem is the **Euler's capital allocation principle**, which states

$$\sum_{j=1}^{N} E_j \frac{\partial \mathrm{VaR}_\alpha}{\partial E_j}(\boldsymbol{E}) = \mathrm{VaR}_\alpha(\boldsymbol{E}), \quad \text{and,} \quad \sum_{j=1}^{N} E_j \frac{\partial \mathrm{ES}_\alpha}{\partial E_j}(\boldsymbol{E}) = \mathrm{ES}_\alpha(\boldsymbol{E}).$$

- The contribution of obligor j to the VaR (ES) at confidence level α,

$$\mathrm{VaRC}_{\alpha,j} := E_j \frac{\partial \mathrm{VaR}_\alpha}{\partial E_j}(\boldsymbol{E}), \quad \text{and,} \quad \mathrm{ESC}_{\alpha,j} := E_j \frac{\partial \mathrm{ES}_\alpha}{\partial E_j}(\boldsymbol{E}).$$

- It can be shown that,

$$\mathrm{VaRC}_{\alpha,j} = \mathbb{E}\left[L_j | L = \mathrm{VaR}_\alpha(\boldsymbol{E})\right], \quad j = 1, \ldots, N,$$

and,

$$\mathrm{ESC}_{\alpha,j} = \mathbb{E}\left[L_j | L \geq \mathrm{VaR}_\alpha(\boldsymbol{E})\right], \quad j = 1, \ldots, N.$$

Non-parametric density estimation by wavelets

- Given i.i.d samples from an unknown statistical distribution X.
- Apply the wavelet theory to approximate the density function f_X.
- We consider the so-called **linear wavelet estimator** (or simply *linear estimator*),

$$f_X(x) \approx \sum_k c_{m,k}\phi_{m,k}(x),$$

  where k varies within a finite range and $\phi_{m,k}(x) = 2^{m/2}\phi(2^m x - k)$. The function ϕ is usually referred to as the **scaling function** or **father wavelet**.
- The coefficients $c_{m,k}$ are, by definition, given by,

$$c_{m,k} := \langle f_X, \phi_{m,k} \rangle = \int_{\mathbb{R}} f_X(x)\bar{\phi}_{m,k}(x)\,\mathrm{d}x = \mathbb{E}\left[\bar{\phi}_{m,k}(X)\right].$$

- The last equality comes from the fact that f_X is a density function.

Application to the loss distribution

- Estimate the density function of the loss variable L by means of the wavelet estimator.
- Generate n samples of the default indicator variable, D_j^i, of obligor j and sample i, where $i = 1, \ldots, n, j = 1 \ldots, N$. Then, $L_j^i = D_j^i \cdot E_j$.
- Denote by L^i the corresponding samples $L^i = \sum_{j=1}^N L_j^i$.
- For convenience, we consider the transformation $Z = \frac{L-a}{b-a}$, and we define $Z^i = \frac{L^i - a}{b-a}, i = 1, \ldots, n$, where,

$$a = \min_{1 \leq i \leq n} \left(L^i \right), \quad b = \max_{1 \leq i \leq n} \left(L^i \right).$$

- From the definition, we can obtain the following unbiased estimator for the wavelet series coefficients,

$$c_{m,k} = \mathbb{E}\left[\bar{\phi}_{m,k}(Z)\right] \approx \frac{1}{n} \sum_{i=1}^{n} \phi_{m,k}(Z^i) =: \hat{c}_{m,k}.$$

Application to the loss distribution

- Using the wavelet density estimation, the unknown density f_L of L can be approximated as follows,

$$f_L(x;\boldsymbol{E}) \approx \hat{f}_L(x;\boldsymbol{E}) := \frac{1}{b-a}\sum_{k=0}^{\mathcal{K}} \hat{c}_{m,k}\phi_{m,k}\left(\frac{x-a}{b-a}\right),$$

  where, by construction, the lower bound for index k is equal to zero and the upper limit is $\mathcal{K} = 2^m - 1$.

- Applying the definition, the distribution function of L can be estimated by

$$\begin{aligned} F_L(x;\boldsymbol{E}) &:= \int_{-\infty}^{x} f_L(y;\boldsymbol{E})\mathrm{d}y \\ &\approx \frac{1}{b-a}\sum_{k=0}^{\mathcal{K}} \hat{c}_{m,k}\int_a^x \phi_{m,k}\left(\frac{y-a}{b-a}\right)\mathrm{d}y =: \hat{F}_L(x;\boldsymbol{E}). \end{aligned}$$

Computation of risk measures

- The VaR value is obtained by using a root-finding method to solve the following equation,

$$\hat{F}_L(x;\boldsymbol{E}) = \alpha,$$

where $\hat{F}_L(x;\boldsymbol{E})$ is the approximation and α is the confidence level.
- Analogously, we use the wavelet approximation of the density, $\hat{f}_L$ in the ES,

$$\mathrm{ES}_\alpha(\boldsymbol{E}) \approx \frac{1}{1-\alpha}\int_{\mathrm{VaR}_\alpha(\boldsymbol{E})}^{b} x\hat{f}_L(x;\boldsymbol{E})\mathrm{d}x,$$

and we get the estimation

$$\mathrm{ES}_\alpha(\boldsymbol{E}) \approx \frac{1}{1-\alpha}\frac{1}{b-a}\sum_{k=0}^{\mathcal{K}} \hat{c}_{m,k}\int_{\mathrm{VaR}_\alpha(\boldsymbol{E})}^{b} x\phi_{m,k}\left(\frac{x-a}{b-a}\right)\mathrm{d}x.$$

- It is worth remarking that the VaR value can be obtained directly from the samples generated by Monte Carlo simulation and the ES can be consequently computed a well.

Computation of risk contributions

- The risk contributions (VaRC and ESC) will be calculated by following the Euler's capital allocation principle.
- Recalling the expression above, the VaR value satisfies,

$$\hat{F}_L(\mathrm{VaR}_\alpha(\boldsymbol{E}); \boldsymbol{E}) = \alpha,$$

- Differentiating we obtain the risk contributions to the VaR

$$\begin{aligned}\mathrm{VaRC}_{\alpha,j} &= E_j \frac{\partial \mathrm{VaR}_\alpha}{\partial E_j}(\boldsymbol{E}) \\ &= -E_j \frac{\frac{\partial \hat{F}_L}{\partial E_j}(\mathrm{VaR}_\alpha(\boldsymbol{E}); \boldsymbol{E})}{\frac{\partial \hat{F}_L}{\partial x}(x; \boldsymbol{E})\big|_{x=\mathrm{VaR}_\alpha(\boldsymbol{E})}} \\ &= -E_j \frac{\frac{\partial \hat{F}_L}{\partial E_j}(\mathrm{VaR}_\alpha(\boldsymbol{E}); \boldsymbol{E})}{\hat{f}_L(\mathrm{VaR}_\alpha(\boldsymbol{E}); \boldsymbol{E})}.\end{aligned}$$

Computation of risk contributions

- If we now integrate by parts the expression for the ES,

$$\mathrm{ES}_\alpha(\boldsymbol{E}) \approx \frac{1}{1-\alpha}\left(b - \alpha \mathrm{VaR}_\alpha(\boldsymbol{E}) - \int_{\mathrm{VaR}_\alpha(\boldsymbol{E})}^{b} \hat{F}_L(x;\boldsymbol{E})\mathrm{d}x\right).$$

- By taking partial derivatives w.r.t. E_j, the risk contributions to the ES are

$$\begin{aligned}\mathrm{ESC}_{\alpha,j} &= E_j \frac{\partial \mathrm{ES}_\alpha}{\partial E_j}(\boldsymbol{E}) \\ &= \frac{1}{1-\alpha} E_j \left(-\alpha \frac{\partial \mathrm{VaR}_\alpha}{\partial E_j}(\boldsymbol{E}) + \frac{\partial \mathrm{VaR}_\alpha}{\partial E_j}(\boldsymbol{E}) \hat{F}_L\left(\mathrm{VaR}_\alpha(\boldsymbol{E});\boldsymbol{E}\right)\right. \\ &\quad \left. - \int_{\mathrm{VaR}_\alpha(\boldsymbol{E})}^{b} \frac{\partial \hat{F}_L}{\partial E_j}(x;\boldsymbol{E})\,\mathrm{d}x\right) \\ &= -\frac{1}{1-\alpha} E_j \int_{\mathrm{VaR}_\alpha(\boldsymbol{E})}^{b} \frac{\partial \hat{F}_L}{\partial E_j}(x;\boldsymbol{E})\,\mathrm{d}x.\end{aligned}$$

Computation of risk contributions

- The VaRC and ESC expressions require the partial derivative of the distribution function w.r.t. the exposures, $\frac{\partial \hat{F}_L(x;\boldsymbol{E})}{\partial E_j}$.
- $\hat{F}_L$ depends on E_j only through the coefficients $\hat{c}_{m,k}$, then

$$\frac{\partial \hat{F}_L}{\partial E_j}(x;\boldsymbol{E}) = \frac{\partial}{\partial E_j}\left(\frac{1}{b-a}\sum_{k=0}^{\mathcal{K}} \hat{c}_{m,k}\int_a^x \phi_{m,k}\left(\frac{y-a}{b-a}\right)\mathrm{d}y\right)$$
$$= \frac{1}{b-a}\sum_{k=0}^{\mathcal{K}} \frac{\partial \hat{c}_{m,k}}{\partial E_j}\int_a^x \phi_{m,k}\left(\frac{y-a}{b-a}\right)\mathrm{d}y.$$

- The partial derivative of the coefficients (assuming ϕ differentiable),

$$\frac{\partial \hat{c}_{m,k}}{\partial E_j} = \frac{\partial}{\partial E_j}\left(\frac{1}{n}\sum_{i=1}^{n}\phi_{m,k}(Z^i)\right) = \frac{1}{n}\sum_{i=1}^{n}\frac{\partial \phi_{m,k}}{\partial E_j}(Z^i)$$
$$= \frac{2^{3m/2}}{b-a}\frac{1}{n}\sum_{i=1}^{n} D_j^i \phi'(2^m Z^i - k).$$

Families of wavelets

- **Haar wavelets**. The Haar scaling function reads,

$$\phi(x) = \begin{cases} 1, & 0 \leq x < 1, \\ 0, & \text{otherwise.} \end{cases}$$

whose "derivative"

$$\phi'(x) = \delta(x) - \delta(x-1) \approx \frac{s}{\pi\left(x^2+s^2\right)} - \frac{s}{\pi\left((x-1)^2+s^2\right)},$$

where δ is the Dirac delta, and $s \to 0$ controls the approximation.

- **Shannon wavelets**. The Shannon scaling function reads,

$$\phi(x) = \mathrm{sinc}(x) = \begin{cases} \sin(\pi x)/(\pi x), & \text{if } x \neq 0, \\ 1, & \text{if } x = 0, \end{cases}$$

where $\mathrm{sinc}(x)$ is usually called *cardinal sine* function. Its derivative is

$$\phi'(x) = \begin{cases} \dfrac{\cos(\pi x)}{x} - \dfrac{\sin(\pi x)}{x^2\pi}, & \text{if } x \neq 0, \\ 0, & \text{if } x = 0. \end{cases}$$

Optimal scale of approximation m

- As usual in density estimation, the optimal convergence rate is achieved by balancing the components within the error.
- The **Mean Integrated Squared Error** (MISE) is commonly employed.
- This error can be split into two terms, **bias** and **variance**, which present an opposite behavior.
- The MISE is defined as,

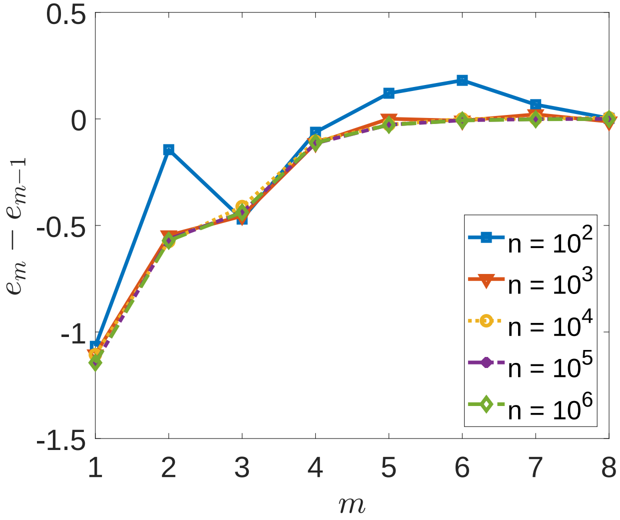
$$\mathrm{MISE} = \int_{\mathbb{R}} \mathbb{E}\left[\left(\hat{f}_L(x;\boldsymbol{E}) - f_L(x)\right)^2\right] \mathrm{d}x,$$

  where $\hat{f}_L$ is the estimated density and f_L is the true density function.
- The difference in the MISE between two consecutive levels of resolution, at scale m (e_m) and at scale $m-1$ (e_{m-1}) is

$$e_m - e_{m-1} \approx \frac{1}{n^2}\sum_{i=1}^{n}\sum_{k=0}^{\mathcal{K}} \phi^2_{m,k}\left(Z^i\right) - \frac{n+1}{n}\sum_{k=0}^{\mathcal{K}}\left(\frac{1}{n}\sum_{i=1}^{n}\psi_{m-1,k}\left(Z^i\right)\right)^2,$$

  where $\psi_{m,k}(x) = 2^{m/2}\psi(2^m x - k)$ and ψ the *mother wavelet*.

Optimal scale of approximation m



Numerical experiments

- Computation of the quantities $\mathrm{VaRC}_{\alpha,j}$ and $\mathrm{ESC}_{\alpha,j}$, $\forall j$, focusing on accuracy, robustness and efficiency of our methodology.
- Computer system characteristics: CPU Intel Core i7-4720HQ 2.6GHz and 16GB RAM.
- The numerical codes have been implemented in C programming language: GNU Scientific Library (GSL).
- The confidence level, α, is set to 99%, and the number of samples is $n = 10^5$, for all the experiments.
- References: WA method [2] and Monte Carlo.
- Two portfolios:

Portfolio	N	P_j	E_j
P1	10000	0.08	$\frac{1}{j}$
P2	25000	0.05	$\frac{1}{j}$

Table: Portfolio configurations.

Comparison: Haar versus Shannon

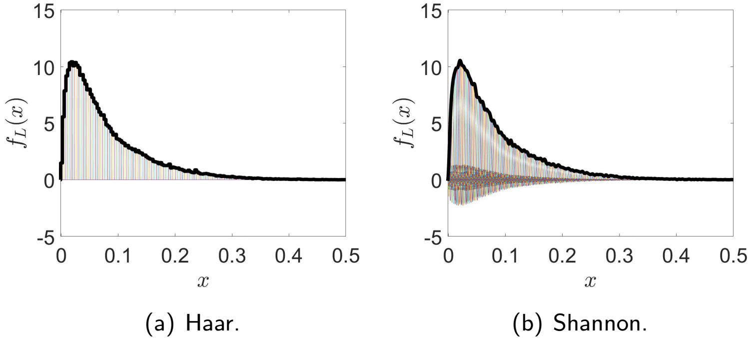
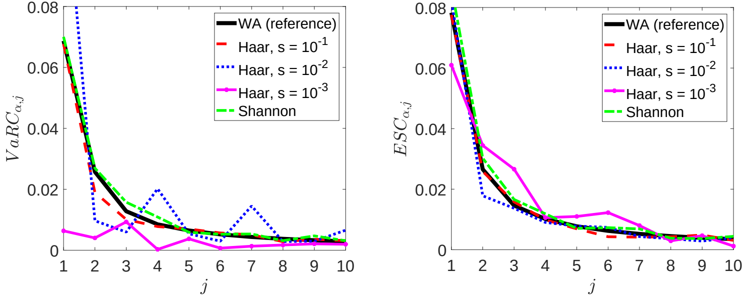


Figure: Estimation of the densities for portfolio P1 with Haar (left plot) and Shannon (right plot).

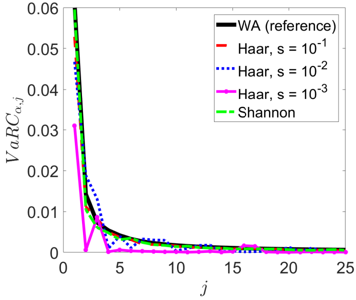
Comparison: Haar versus Shannon



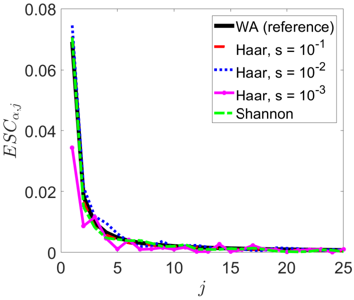
(a) VaR contributions. (b) ES contributions.

Figure: Portfolio P1: risk contributions ($j = 1, \ldots, 10$).

Comparison: Haar versus Shannon



(a) VaR contributions.



(b) ES contributions.

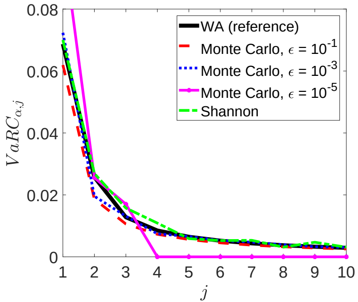
Figure: Portfolio P2: risk contributions ($j = 1, \ldots, 25$).

Comparison: Haar versus Shannon

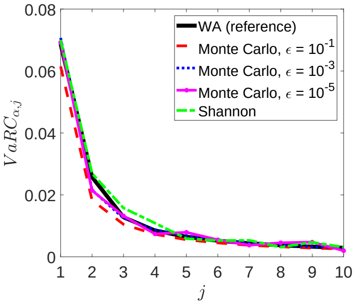
	Portfolio P1		Portfolio P2	
	$\sum \mathrm{VaRC}_{\alpha,j}$	$\sum \mathrm{ESC}_{\alpha,j}$	$\sum \mathrm{VaRC}_{\alpha,j}$	$\sum \mathrm{ESC}_{\alpha,j}$
WA ($m = 10$)	0.3227	0.3658	0.2153	0.2429
Haar ($s = 0.1$)	0.2667	0.3440	0.1847	0.2315
Haar ($s = 0.01$)	0.4016	0.4684	0.1762	0.2799
Haar ($s = 0.001$)	0.6411	0.4236	0.0753	0.1599
Shannon	0.3236	0.3681	0.2091	0.2457

Table: Influence of the steepness parameter s, in the Haar-based data-driven approximation.

Comparison: Shannon versus crude Monte Carlo simulation



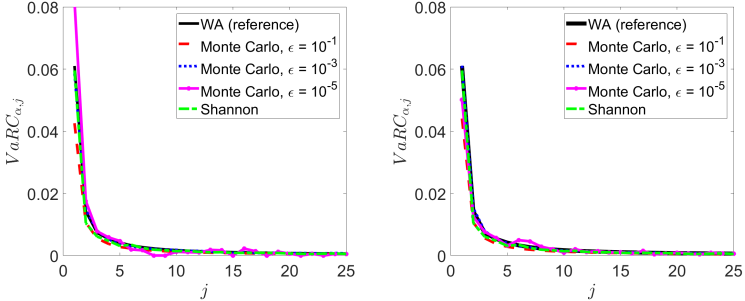
(a) $n_{\mathrm{MC}} = 10^6$.



(b) $n_{\mathrm{MC}} = 10^7$.

Figure: VaR contributions (VaRC) with Monte Carlo varying ϵ. Portfolio P1.

Comparison: Shannon versus crude Monte Carlo simulation

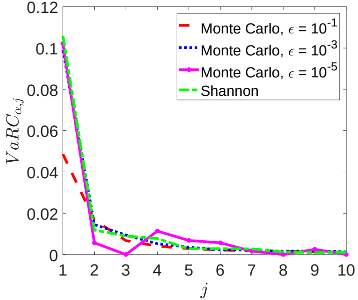


(a) $n_{\text{MC}} = 10^6$.

(b) $n_{\text{MC}} = 10^7$.

Figure: VaR contributions (VaRC) with Monte Carlo varying ϵ. Portfolio P2.

Experiments on multi-factor models



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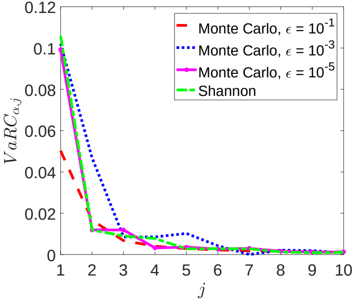
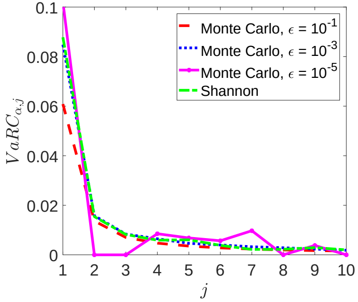


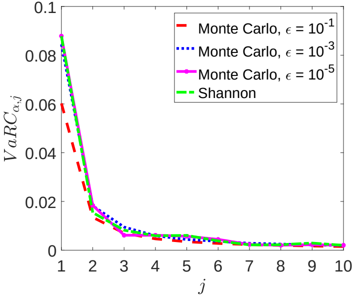
(b) $n_{\mathrm{MC}} = 10^7$.

Figure: Multi-factor Gaussian copula: VaR contributions portfolio P1 and $d = 5$.

Experiments on multi-factor models



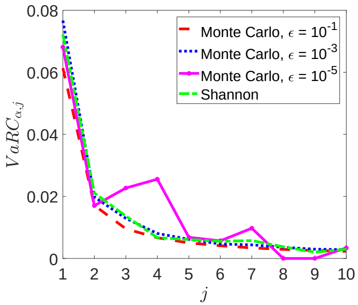
(a) $n_{MC} = 10^6$.



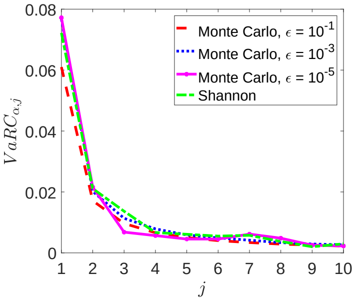
(b) $n_{MC} = 10^7$.

Figure: Multi-factor Gaussian copula: VaR contributions for portfolio P1 and $d = 25$.

Experiments on multi-factor models



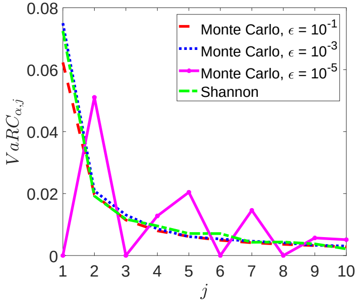
(a) $n_{MC} = 10^6$.



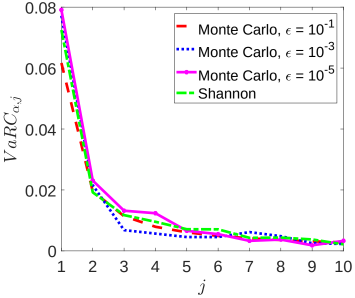
(b) $n_{MC} = 10^7$.

Figure: Multi-factor t-copula: VaR contributions for portfolio P1 and $d = 5$.

Experiments on multi-factor models



(a) $n_{MC} = 10^6$.



(b) $n_{MC} = 10^7$.

Figure: Multi-factor t-copula: VaR contributions for portfolio P1 and $d = 25$.

Computational performance

		$d = 5$ ($m = 7$)		$d = 25$ ($m = 7$)	
Method	Samples	Time	Speed-up	Time	Speed-up
Shannon	$n = 10^5$	90	$\times 1$	91	$\times 1$
MC	$n_{\text{MC}} = 10^6$	3330	$\times 37$	9759	$\times 107$
MC	$n_{\text{MC}} = 10^7$	33260	$\times 370$	99252	$\times 1091$

Table: Time and speed-up: multi-factor Gaussian copula model. Portfolio P1.

		$d = 5$ ($m = 8$)		$d = 25$ ($m = 8$)	
Method	Samples	Time	Speed-up	Time	Speed-up
Shannon	$n = 10^5$	210	$\times 1$	213	$\times 1$
MC	$n_{\text{MC}} = 10^6$	3181	$\times 15$	10376	$\times 49$
MC	$n_{\text{MC}} = 10^7$	33135	$\times 158$	99932	$\times 469$

Table: Time and speed-up: multi-factor t-copula model. Portfolio P2.

Conclusions

- We have investigated the computation of risk contributions to VaR and ES in a credit portfolio by means of non-parametric density estimation based on wavelets, particularly Haar and Shannon.
- While the Haar family has desirable properties like compact support and positiveness, we finally prefer the Shannon family due to its robustness and easy handling.
- We have intensively tested our method, considering one- and multi-factor Gaussian and t-copula models and two different portfolios.
- Our methodology turns out to be a robust, accurate and efficient alternative to Monte Carlo methods, commonly used in practice.
- To the best of our knowledge, this is the first time that this approach is followed for solving the capital allocation problem by means of Euler's capital allocation principle.

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Acknowledgements & Questions

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Multi-factor models

- The incentive for considering the multi-factor version of the Gaussian copula model becomes clear when one rewrites it in matrix form,

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix} = \begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{N1} \end{bmatrix} Y_1 + \begin{bmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{N2} \end{bmatrix} Y_2 + \cdots + \begin{bmatrix} a_{1d} \\ a_{2d} \\ \vdots \\ a_{Nd} \end{bmatrix} Y_d + \begin{bmatrix} b_1\varepsilon_1 \\ b_2\varepsilon_2 \\ \vdots \\ b_N\varepsilon_N \end{bmatrix}.$$

- While each ε_j represents the idiosyncratic factor affecting only obligor j, the common factors $Y_1, Y_2 \ldots, Y_d$, may affect all (or a certain group of) obligors.
- Although the systematic factors are sometimes given economic interpretations (as industry or regional risk factors, for example), their key role is that they allow us to model complicated correlation structures in a non-homogeneous portfolio.

Mother wavelets

- Here we present the definition of the mother wavelet functions for both Haar and Shannon families. Thus, in the case of Haar basis, the mother wavelet reads,

$$\psi(x) := \begin{cases} 1, & 0 \leq x < \frac{1}{2}, \\ -1, & \frac{1}{2} \leq x < 1, \\ 0, & \text{otherwise}, \end{cases}$$

while the Shannon mother wavelet is defined as,

$$\psi(x) := \frac{\sin\left(\pi\left(x - \frac{1}{2}\right)\right) - \sin\left(2\pi\left(x - \frac{1}{2}\right)\right)}{\pi\left(x - \frac{1}{2}\right)} = 2\mathrm{sinc}(2x-1) - \mathrm{sinc}(x).$$